

Supplementary Appendix: Treatment Effects in Interactive Fixed Effects Models

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This supplementary appendix contains (i) additional identification results for ATT_t when one is willing to make the additional assumption that the time varying unobservables in the model for untreated potential outcomes are serially uncorrelated or that there are strictly exogenous time varying covariates that potentially provide an additional source of moment conditions that were not available in the case considered in the main text; (ii) proofs for Theorem 2 and Propositions 1 to 4 from the main text; (iii) results from some additional Monte Carlo simulations on model selection with a weak instrument; and (iv) some additional results for the application on job displacement considered in the main text.

SA-1 Alternative Approaches to Identification

In this section, we briefly consider some other possible approaches to identifying the ATT with a small number of time periods. In particular, we consider two additional approaches: (i) assuming that the time varying unobservables in the model in Assumption 1 are serially uncorrelated, and (ii) assuming access to time varying covariates in combination with a strict exogeneity condition. In principle, either approach can be used by itself or in combination with our main approach or with each other to identify the ATT.

Identification through serially uncorrelated unobservables

One common approach is to assume that the time varying unobservables, U_t in the model in Assumption 1, are mutually independent (or at least serially uncorrelated). This is a fairly common assumption in the literature on interactive fixed effects models.¹ These types of assumptions are also commonly made in the related literature on factor models with repeated measurements.

In this setup, outcomes themselves can be correlated over time, but the correlation is only allowed through the time-invariant unobservables ξ and λ . Ruling out serial correlation in U_t is likely to be more plausible when one includes interactive fixed effects, but the overall plausibility of this sort of assumption is likely to be application specific, and it is one that researchers should think carefully about. For example, this kind of restriction is incompatible with the common practice of computing standard errors that are robust to serial correlation (e.g., Bertrand, Duflo, and Mullainathan (2004)) in difference in differences or linear trend models. The approach outlined in this subsection requires

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¹See Freyberger (2018, Section 2.1) for a recent discussion of this type of identification argument in the context of panel data models (though not treatment effect models).

four time periods. Three time periods are required for the “differencing” approach discussed in the main text. Another extra period of outcomes is required to use an outcome in a different time period as an instrument. We also require having access to panel data; i.e., repeated cross sections data will not be suitable for this approach. On the other hand, this approach does not require that the effect of some time invariant covariate not change over time.

Next, we introduce several more conditions required for identification of the ATT in this case.

Assumption U1 (Serially Uncorrelated Time-Varying Unobservables). *The U_t are serially uncorrelated; i.e., $\mathbb{E}[U_t U_s | D = d] = 0$ for all $t \neq s$ and $d \in \{0, 1\}$.*

Assumption **U1** rules out serial correlation in the time varying unobservables. Our identification argument proceeds as follows. As before, least squares estimates of (β_t^*, F_t^*) are biased because $(Y_{it^*-1} - Y_{it^*-2})$ is (by construction) correlated with V_{it} . However, under Assumption **U1** and after differencing out the fixed effects as in Equation (5), Y_{is} (with $s \in \{1, 2, \dots, \mathcal{T}\} \setminus \{t, t^* - 1, t^* - 2\}$) can be used as an instrument. In particular, the following moment conditions hold:

$$\mathbb{E}[ZV_{it}|D = 0] = 0 \quad \text{and} \quad \mathbb{E}[Y_{is}V_{it}|D = 0] = 0 \quad \text{for } s \in \{1, 2, \dots, \mathcal{T}\} \setminus \{t, t^* - 1, t^* - 2\} \quad (\text{SA-1})$$

For any $t \in \{t^*, \dots, \mathcal{T}\}$, Equation (SA-1) provides $K + (\mathcal{T} - 3)$ moment equalities from the untreated group. Thus, as before, for any $t \in \{t^*, \dots, \mathcal{T}\}$, there are $K + 1$ parameters to identify, and the order condition is satisfied as long as there are at least four time periods of available data. Also, notice that this approach does not require the availability of any time invariant covariates whose effect does not change over time though if these are available, one can still incorporate these extra restrictions.

Next, we introduce a bit more notation. Let $Y_{-t,t'} = (Y_1, Y_2, \dots, Y_{t'})' \setminus \{Y_t, Y_{t^*-1}, Y_{t^*-2}\}$; i.e, the vector of outcomes across all time periods from time period 1 to t' excluding time periods t , $t^* - 1$, and $t^* - 2$. Next, define $\tilde{Z}_t = (Z', Y'_{-t,\mathcal{T}})'$ which is a $K + (\mathcal{T} - 3) \times 1$ vector of exogenous variables for period t .

Assumption U2 (Timing). $\mathcal{T} \geq 4$ and $t^* \geq 3$ (i.e., there are at least four total time periods and at least two of these are pre-treatment).

Assumption U3 (Relevance). *For $t = t^*, \dots, \mathcal{T}$, the matrix $\mathbf{M}_{2t} := \mathbb{E} \left[(1 - D) \tilde{Z}_t (Z', (Y_{t^*-1} - Y_{t^*-2})) \right]$ has full rank.*

The data requirements in Assumption **U2** are strictly stronger than those required in a difference in differences type setup (typically two periods) and in models with individual-specific linear trends (typically three periods) as well as in our approach considered in the main text. Assumption **U3** is a relevance condition for using outcomes in other periods as instruments. The main requirement is that $Y_{-t,\mathcal{T}}$ is correlated with $(Y_{t^*-1} - Y_{t^*-2})$ after controlling for Z . This will hold if $Y_{-t,\mathcal{T}}$ is correlated with λ after controlling for Z . This is an instrument relevance condition that can be checked with the available data.

Proposition SA-1. *Under Assumptions 1, 2, 4, 5, U1, U2, and U3, β_t^* and F_t^* are identified for all $t = t^*, \dots, \mathcal{T}$. In particular, for any $m_2 \times m_2$ (with $m_2 := K + \mathcal{T} - 3$) positive definite weighting matrix \mathbf{W} ,*

$$(\beta_t^*, F_t^*)' = (\mathbf{M}'_{2t} \mathbf{W} \mathbf{M}_{2t})^{-1} \mathbf{M}'_{2t} \mathbf{W} \mathbb{E}[(1 - D) \tilde{Z}_t (Y_t - Y_{t^*-2})]$$

In addition, ATT_t is identified for all $t = t^, \dots, \mathcal{T}$, and it is given by*

$$ATT_t = \mathbb{E}[Y_t - Y_{t^*-2} | D = 1] - \left(\mathbb{E}[X' | D = 1] \beta_t^* + F_t^* (\mathbb{E}[Y_{t^*-1} - Y_{t^*-2} | D = 1]) \right)$$

Proof. The result holds immediately using the same arguments as in the proof of Theorem 1. \square

Remark SA-1. *There are alternative approaches that could be considered under the same assumptions. For example, one could use the covariance matrix of outcomes over time for the untreated group and pre-treatment outcomes for the treated group to identify the parameters of the model. See Williams (2020) as a recent, relevant example in a different context though noting that similar arguments are likely to apply in our setting as well.*

Finally, it is useful to compare our results in this section to existing results for factor models that use similar independence conditions to obtain identification. The setup of the model in Assumption 1 combined with assumptions in this subsection is quite similar to a factor model with two factors. A typical setup there would require independence of the unobservables across equations (corresponding to our condition that the time varying unobservables be uncorrelated with each other as in Assumption U1). That case typically requires five “measurements” (corresponding to our number of time periods). The number of time periods in our case is reduced by one because the coefficient on one of the time invariant unobservables, ξ , is restricted not to change over time. Thus, these results are quite similar. More interestingly though, in our case, even when there are only four time periods, either ATT_3 is identified (this happens if $t^* = 3$) or there are testable implications of our approach (this happens if $t^* = 4$). The reason for this is that we can identify $(\beta_3^*, F_3^*, \beta_4^*, F_4^*)$ only using information from the untreated group.

Identification through time varying covariates

To extend the model in Assumption 1 to the case with time-varying covariates, consider the following model for untreated potential outcomes

$$Y_{it}(0) = \theta_t + \xi_i + \lambda_i F_t + X'_{it} \beta + U_{it} \tag{SA-2}$$

In principle, it is straightforward to accommodate both time invariant and time varying covariates simultaneously, but, for simplicity, we consider the case where there are only time varying covariates here. Following essentially the same differencing arguments as in the main text, we can write

$$Y_{it}(0) - Y_{it^*-2} = \theta_t^* + (X_{it} - X_{it^*-2})' \beta + F_t^* (Y_{it^*-1} - Y_{it^*-2}) - (X_{it^*-1} - X_{it^*-2})' \beta F_t^* + V_{it}$$

where $V_{it} = (U_{it} - U_{it^*-2}) - F_t^* (U_{it^*-1} - U_{it^*-2})$. Consider the following assumption.

Assumption X1 (Strict Exogeneity). *Let $X^T = (1, X'_1, \dots, X'_T)'$. For all $(t, d) \in \{1, \dots, T\} \times \{0, 1\}$, $\mathbb{E}[U_t | X^T, \xi, \lambda, D = d] = 0$.*

Assumption X1 is a common strict exogeneity assumption for time varying covariates in the context of panel data models. It implies that U_t (the time varying unobservables) are uncorrelated with the covariates in all time periods. It seems possible to extend the arguments in this section to alternative assumptions on the covariates such as pre-determinedness (though this might require adjusting the differencing strategy that we have been using throughout the paper); see, for example, Arellano and Honoré (2001) for discussion of these types of assumptions. Assumption X1 allows us to use covariates in periods besides period t as instruments for $(Y_{t^*-1} - Y_{t^*-2})$. Finally, we make assumptions about the sampling scheme, treatment timing, and a relevance assumption.

Assumption X2 (Observed Data). *The observed data consists of iid draws of $\{Y_{i1}, Y_{i2}, \dots, Y_{iT}, X_{i1}, X_{i2}, \dots, X_{iT}, D_i\}_{i=1}^n$, where n denotes the sample size.*

Assumption X3 (Timing). *$T \geq 3$ and $t^* \geq 3$ (i.e., there are at least three total time periods and at least two pre-treatment time periods).*

Assumption X4 (Relevance). For $t = t^*, \dots, \mathcal{T}$, the matrix $\mathbf{M}_{\mathbf{x}_t} := \mathbb{E}[(1 - D)X^\mathcal{T}(1, (X_t - X_{t^*-2})', Y_{t^*-1} - Y_{t^*-2}, (X_{t^*-1} - X_{t^*-2})')]$ has full rank.

Assumption X2 extends the iid panel sampling scheme to allow for time varying covariates. Assumption X3 is the same as Assumption 3 in the main text. Assumption X4 is an instrument relevance condition for using covariates in alternative time periods as an instrument to identify F_t^* .

Proposition SA-2. In the model in Equation (SA-2) and under Assumptions 4, X1, X2, X3, and X4, β_t^* and F_t^* are identified for all $t = t^*, \dots, \mathcal{T}$. In particular, for any $m_3 \times m_3$ (with m_3 the number of rows of $\mathbf{M}_{\mathbf{x}_t}$) positive definite weighting matrix \mathbf{W} ,

$$(\theta_t^*, \beta', F_t^*, \zeta_t')' = (\mathbf{M}'_{\mathbf{x}_t} \mathbf{W} \mathbf{M}_{\mathbf{x}_t})^{-1} \mathbf{M}'_{\mathbf{x}_t} \mathbf{W} \mathbb{E}[(1 - D)X^\mathcal{T}(Y_t - Y_{t^*-2})]$$

where ζ_t is an extra nuisance parameter that is equal to βF_t^* (though we consider it separately here). In addition, ATT_t is identified for all $t = t^*, \dots, \mathcal{T}$, and it is given by

$$ATT_t = \mathbb{E}[Y_t - Y_{t^*-2} | D = 1] - \left(\theta_t^* + \mathbb{E}[(X_t - X_{t^*-2})' | D = 1] \beta + F_t^* (\mathbb{E}[Y_{t^*-1} - Y_{t^*-2} | D = 1]) - \mathbb{E}[(X_{t^*-1} - X_{t^*-2})' | D = 1] \zeta_t \right)$$

Proof. The result holds immediately using the same arguments as in the proof of Theorem 1. \square

To conclude this section, it is worth making a few comments about the sort of model in Equation (SA-2). First (and momentarily ignoring the interactive fixed effect term), there are some important conceptual issues with this sort of model in the context of policy evaluation. In the presence of time varying covariates, work in the econometrics literature on difference in differences where the parallel trends assumption holds after conditioning on covariates (e.g., Heckman, Ichimura, Smith, and Todd (1998), Abadie (2005), and Callaway and Sant'Anna (2021)) typically conditions on a particular pre-treatment value of the covariates. This sort of setup uses the group of individuals who do not participate in the treatment and have the same value of the pre-treatment covariates as the comparison group. This is consistent with the approach in the current paper of including a pre-treatment value of a covariate that varies over time but allowing its effect to change over time. On the other hand, the model in Equation (SA-2) effectively uses individuals whose covariates *change* by the same amount over time (though whose level could be quite different) as the comparison group. This does not seem to be the interpretation that most applied work is aiming for. A second main issue with the approach in this section is that, even if the model in Equation (SA-2) is correct (and hence β and F_t^* can be identified), covariates that can vary over time may be affected by participating in the treatment. In this case, it would probably make sense to define treated and untreated potential covariates (these sorts of points are made in Lechner (2008) and Bonhomme and Sauder (2011)). Then, identifying the ATT would require identifying the average of untreated potential covariates for the treated group. However, this term is not immediately identified and would likely require a number of additional assumptions and perhaps introduce other complications as well.

SA-2 Additional Proofs

In this section, we provide the proofs for Theorem 2 and Propositions 1 to 4 from the main text.

Proof of Theorem 2

To start with, we prove an intermediate result providing an asymptotically linear representation for the first stage estimates coming from the interactive fixed effects model for untreated potential outcomes.

Lemma SA-2.1. *Under Assumptions 1 to 8 and for any $t \in \{t^*, \dots, \mathcal{T}\}$,*

$$\sqrt{n} \begin{pmatrix} \hat{\beta}_t^* - \beta_t^* \\ \hat{F}_t^* - F_t^* \end{pmatrix} = \frac{1}{\sqrt{n}} \sum_{i=1}^n \varphi_t^{GMM}(D_i, Z_i, Y_i) + o_p(1)$$

Proof. Start by defining

$$\varphi_\gamma(D_i, Z_i, Y_i) := (\mathbb{E}[\mathbf{Z}\mathbf{X}]' \mathbf{W} \mathbb{E}[\mathbf{Z}\mathbf{X}])^{-1} \mathbb{E}[\mathbf{Z}\mathbf{X}]' \mathbf{W} \mathbf{Z}_i V_i$$

Standard arguments on GMM immediately imply that

$$\sqrt{n}(\hat{\gamma} - \gamma) = \frac{1}{\sqrt{n}} \sum_{i=1}^n \psi_\gamma(D_i, Z_i, Y_i) + o_p(1) \tag{SA-3}$$

To complete the proof, notice that

$$\begin{aligned} \sqrt{n} \begin{pmatrix} \hat{\beta}_t^* - \beta_t^* \\ \hat{F}_t^* - F_t^* \end{pmatrix} &= \Xi_t \sqrt{n}(\hat{\gamma} - \gamma) \\ &= \Xi_t \frac{1}{\sqrt{n}} \sum_{i=1}^n \psi_\gamma(D_i, Z_i, Y_i) + o_p(1) \\ &= \frac{1}{\sqrt{n}} \sum_{i=1}^n \varphi_t^{GMM}(D_i, Z_i, Y_i) + o_p(1) \end{aligned}$$

where the first equality holds immediately, the second equality holds by Equation (SA-3), and the last equality holds by the definition of φ_t^{GMM} . \square

Proof of Theorem 2. First, notice that

$$\sqrt{n}(\hat{p} - p) = \frac{1}{\sqrt{n}} \sum_{i=1}^n \varphi^P(D_i)$$

Next, notice that

$$\begin{aligned}
& \sqrt{n} \left(\widehat{ATT}_t - ATT_t \right) \\
&= \sqrt{n} (\hat{p}^{-1} - p^{-1}) \left(\frac{1}{n} \sum_{i=1}^n D_i (Y_{it} - Y_{it^*-2}) - \frac{1}{n} \sum_{i=1}^n D_i X_i' \hat{\beta}_t^* - \hat{F}_t^* \frac{1}{n} \sum_{i=1}^n D_i (Y_{it^*-1} - Y_{it^*-2}) \right) \\
&\quad + p^{-1} \frac{1}{\sqrt{n}} \sum_{i=1}^n \varphi_t^M(D_i, X_i, Y_i)' \begin{pmatrix} 1 \\ \hat{\beta}_t^* \\ \hat{F}_t^* \end{pmatrix} \\
&\quad + p^{-1} \left[\begin{pmatrix} \mathbb{E}[DX] \\ \mathbb{E}[D(Y_{t^*-1} - Y_{t^*-2})] \end{pmatrix}' \sqrt{n} \begin{pmatrix} \hat{\beta}_t^* - \beta_t^* \\ \hat{F}_t^* - F_t^* \end{pmatrix} \right] \\
&:= A + B + C
\end{aligned}$$

which follows by the expressions for \widehat{ATT}_t and ATT_t in Equations (16) and (17), by adding and subtracting several terms, and by the definition of φ_t^M . We consider A , B , and C in turn.

First, consider term A .

$$\begin{aligned}
A &= -\frac{\sqrt{n}(\hat{p} - p)}{\hat{p}p} \left(\frac{1}{n} \sum_{i=1}^n D_i (Y_{it} - Y_{it^*-2}) - \frac{1}{n} \sum_{i=1}^n D_i X_i' \hat{\beta}_t^* - \hat{F}_t^* \frac{1}{n} \sum_{i=1}^n D_i (Y_{it^*-1} - Y_{it^*-2}) \right) \\
&= -\frac{ATT_t}{p^2} \frac{1}{\sqrt{n}} \sum_{i=1}^n \varphi_t^P(D_i) + o_p(1) \\
&= \frac{1}{\sqrt{n}} \sum_{i=1}^n \psi_t^A(D_i) + o_p(1)
\end{aligned}$$

where the first equality holds by the definition of term A and by multiplying and dividing the first term by p and the second term by \hat{p} and then combining terms, the second equality holds by the weak law of large numbers and continuous mapping theorem, and the last equality holds by the definition of ψ_t^A .

Next, consider term B .

$$B = \frac{1}{\sqrt{n}} \sum_{i=1}^n \psi_t^B(D_i, X_i, Y_i) + o_p(1)$$

which immediately holds from the definition of term B because $\hat{\beta}_t^*$ and \hat{F}_t^* are consistent for β_t^* and F_t^* and by the definition of ψ_t^B .

Finally, consider term C . From Lemma SA-2.1 and the continuous mapping theorem, it immediately holds that

$$\begin{aligned}
C &= p^{-1} \left(\begin{pmatrix} \mathbb{E}[DX] \\ \mathbb{E}[D(Y_{t^*-1} - Y_{t^*-2})] \end{pmatrix}' \frac{1}{\sqrt{n}} \sum_{i=1}^n \varphi_t^{GMM}(D_i, Z_i, Y_i) + o_p(1) \right) \\
&= \frac{1}{\sqrt{n}} \sum_{i=1}^n \psi_t^C(D_i, Z_i, Y_i) + o_p(1)
\end{aligned}$$

Combining the expressions for terms A , B , and C implies that, for any $t \in \{t^*, \dots, \mathcal{T}\}$

$$\sqrt{n} \left(\widehat{ATT}_t - ATT_t \right) = \frac{1}{\sqrt{n}} \sum_{i=1}^n \psi_t(D_i, Z_i, Y_i) + o_p(1) \quad (\text{SA-4})$$

which holds by the definition of ψ_t and completes the first part of the proof. The second part of the proof holds by applying the central limit theorem jointly to the above expression for all $t = t^*, \dots, \mathcal{T}$. \square

Proofs of Propositions 1 to 4

The results in Propositions 1, 3, and 4 all hold using essentially the same arguments as in Theorem 1. For Proposition 2, the proof is also similar but we need to account for the repeated cross sections sampling scheme in Assumption RC.

Proof of Proposition 2. First, the validity of the moment conditions in Equation (22) holds immediately from Assumption RC. This implies the result in the first part of the proposition concerning identification of γ (the parameters from the interactive fixed effects model for untreated potential outcomes). For the second part of the result, notice that

$$\begin{aligned}
ATT_t &= \mathbb{E}[Y_t - Y_{t^*-2} | D = 1] - \left(\mathbb{E}[X' | D = 1] \beta_t^* + F_t^* \mathbb{E}[Y_{t^*-1} - Y_{t^*-2} | D = 1] \right) \\
&= \left(\mathbb{E}_M[Y | D = 1, T = t] - \mathbb{E}_M[Y | D = 1, T = t^* - 2] \right) \\
&\quad - \left(\mathbb{E}_M[X' | D = 1] \beta_t^* + F_t^* \left(\mathbb{E}_M[Y | D = 1, T = t^* - 1] - \mathbb{E}_M[Y | D = 1, T = t^* - 2] \right) \right) \\
&= \mathbb{E}_M \left[\frac{T_t}{\pi_t} Y - \frac{T_{t^*-2}}{\pi_{t^*-2}} Y \mid D = 1 \right] \\
&\quad - \left\{ \mathbb{E}_M[X' | D = 1] \beta_t^* + F_t^* \left(\mathbb{E}_M \left[\frac{T_{t^*-1}}{\pi_{t^*-1}} Y - \frac{T_{t^*-2}}{\pi_{t^*-2}} Y \mid D = 1 \right] \right) \right\}
\end{aligned}$$

where the first equality holds from the same arguments as in Theorem 1, and second and third equalities hold by Assumption RC. \square

SA-3 Additional Monte Carlo Simulations

This section contains an additional set of Monte Carlo simulations related to selecting the number of interactive fixed effects in the model for untreated potential outcomes when the instrument is weak.

Table SA-1: Model Selection with Weak Instrument

	BIC(0)	BIC(1)	BIC(2)	BIC(3)	CV-U(0)	CV-U(1)	CV-U(2)	CV-U(3)	CV-T(0)	CV-T(1)	CV-T(2)	CV-T(3)
<i>IFE = 0</i>												
$n = 100$	1	0	0	0	0.476	0.418	0.097	0.009	0.365	0.392	0.195	0.048
$n = 500$	1	0	0	0	0.360	0.399	0.214	0.027	0.342	0.393	0.217	0.048
$n = 1000$	1	0	0	0	0.354	0.438	0.174	0.034	0.336	0.426	0.197	0.041
<i>IFE = 1</i>												
$n = 100$	0.999	0.001	0	0	0.926	0.033	0.029	0.012	0.237	0.275	0.303	0.185
$n = 500$	0.999	0.001	0	0	0.882	0.013	0.071	0.034	0.120	0.336	0.352	0.192
$n = 1000$	0.995	0.005	0	0	0.852	0.003	0.077	0.068	0.059	0.367	0.369	0.205
<i>IFE = 2</i>												
$n = 100$	0.029	0.827	0.141	0.003	0.013	0.523	0.398	0.066	0.001	0.330	0.396	0.273
$n = 500$	0.007	0.677	0.312	0.004	0	0.315	0.506	0.179	0	0.231	0.490	0.279
$n = 1000$	0	0.425	0.568	0.007	0	0.265	0.518	0.217	0	0.183	0.526	0.291
<i>IFE = 3</i>												
$n = 100$	0.040	0.144	0.319	0.497	0.141	0.361	0.275	0.223	0.451	0.039	0.170	0.340
$n = 500$	0.009	0.038	0.210	0.743	0.019	0.192	0.180	0.609	0.249	0.012	0.148	0.591
$n = 1000$	0.002	0.009	0.069	0.920	0.004	0.074	0.125	0.797	0.125	0.004	0.107	0.764

Notes: The table provides estimates of the fraction of times that different model selection criteria (BIC, cross validation using the untreated group, and cross validation using the treated group) chose different numbers of interactive fixed effects in the model for untreated potential outcomes. Results are computed using 1000 Monte Carlo simulations and vary the true number of interactive fixed effects and number of observations across simulations. Panels in the table are separated by the true number of interactive fixed effects in the model for untreated potential outcomes (e.g., in the top panel, labeled “ $IFE = 0$ ”, the true number of interactive fixed effects is 0). Within each panel, the number of observations varies among 100, 500, and 1000. Columns labeled “BIC(j)” provide the fraction of times that model j was chosen using the BIC model selection criteria. Columns labeled “CV-U(j)” provide the fraction of times that model j was chosen using cross validation among the untreated group (as discussed in the text). Columns labeled “CV-T(j)” provide the fraction of times that model j was chosen using cross validation among the treated group (as discussed in the text). Thus, for example, the value 0.476 in the first row and column labeled “CV-U(0)” indicates that cross validation using the untreated group selected the model with zero interactive fixed effects in 476 out of 1000 simulations when the true number of interactive fixed effects was zero and there were 100 observations.

As discussed in the main text, the overall performance of our model selection procedure in the presence of weak instruments is poor, but this is a somewhat unusual case as a researcher would likely be able to detect a weak instrument problem and, therefore, be unlikely to try to choose the number of interactive fixed effects in this scenario.

SA-4 Additional Results on Job Displacement

This section contains some additional results for the application in the main text about job displacement. In particular, it provides estimates for the interactive fixed effects model for untreated potential outcomes when (i) education is used as the covariate with time invariant effects, (ii) AFQT is used as the covariate with time invariant effects and additional covariates are included in the model, and (iii) education is used as the covariate with time invariant effects and additional covariates are included in the model. In addition, it contains the complete first stage estimates for the main case considered in the main text where AFQT is used as the covariate with time invariant effects and no additional covariates are included.

Table SA-2: IFE Model Estimates using Education as Covariate with Time Invariant Effect

(a) Pre-Treatment Periods

	g:89,t:87	g:91,t:87	g:91,t:89	g:93,t:87	g:93,t:89	g:93,t:91
(Intercept)	-1.48 (0.99)	-1.46 (1.02)	-0.73 (0.86)	-1.84 (1.12)	-0.38 (0.84)	-1.14 (0.92)
IFE	2.32* (0.25)	2.31* (0.25)	2.21* (0.22)	2.42* (0.28)	2.12* (0.22)	2.10* (0.23)
N	2721	2567	2567	2588	2434	2434
Sargan p-value	0.527	0.677	0.589	0.784	0.528	0.802
Weak IV F-stat	29.09	27.59	33.65	23.45	32.54	29.58

* $p < 0.05$

(b) Post-Treatment Periods

	g:89,t:89	g:89,t:91	g:89,t:93	g:91,t:91	g:91,t:93	g:93,t:93
(Intercept)	-4.00* (1.86)	-8.10* (3.05)	-8.17* (3.42)	-1.95 (1.47)	-1.23 (1.66)	-0.17 (1.13)
IFE	3.91* (0.46)	5.73* (0.76)	6.35* (0.86)	3.36* (0.38)	3.80* (0.43)	2.47* (0.28)
N	2567	2434	2434	2434	2434	2434
Sargan p-value	0.869	0.719	0.995	0.554	0.932	0.341
Weak IV F-stat	27.59	23.45	23.45	32.54	32.54	29.58

* $p < 0.05$

Notes: The table contains estimates of the interactive fixed effects model for untreated potential outcomes using educational attainment (this amounts to dummy variables indicating being a college graduate or being a high school graduate) as the covariates whose effects on untreated potential outcomes do not change over time and without including other covariates. Columns correspond to estimates for particular groups and particular time periods. For example, the first column in panel (a), which is labeled “g:89,t:87” is for the group of workers who were displaced in 1989 in the year 1987. The rows labeled “IFE” report the estimated value of F_t^* . The row labeled “Sargan p-value” provides the p-value from an over-identification test. The row labeled “Weak IV F-stat” report the F-statistic from the first stage regression of the endogenous regressors on educational attainment. Some F-statistics are the same because the first stage is the same across some groups/time periods.

Table SA-3: IFE Model Estimates using AFQT as Covariate with Time Invariant Effect and Additional Covariates

(a) Pre-treatment periods

	g:89,t:87	g:91,t:87	g:91,t:89	g:93,t:87	g:93,t:89	g:93,t:91
(Intercept)	-1.49 (1.53)	-1.51 (1.60)	1.09 (1.28)	-1.23 (1.59)	1.43 (1.37)	-0.06 (2.33)
High School	0.71 (0.90)	0.52 (0.98)	0.22 (1.12)	0.32 (1.03)	-0.06 (1.17)	0.69 (1.40)
College	1.27 (1.36)	1.03 (1.45)	1.64 (1.82)	1.10 (1.47)	1.07 (1.93)	1.76 (2.92)
Black	0.74 (0.72)	0.83 (0.76)	-0.41 (0.79)	0.65 (0.78)	-0.49 (0.84)	-1.32 (1.02)
White	0.44 (0.62)	0.39 (0.65)	0.12 (0.69)	0.28 (0.68)	-0.03 (0.74)	-1.08 (0.89)
Female	0.07 (0.89)	0.20 (0.96)	-1.17 (0.94)	0.20 (0.98)	-1.18 (0.99)	0.22 (1.88)
IFE	1.98* (0.34)	2.02* (0.35)	1.70* (0.37)	1.99* (0.36)	1.75* (0.39)	1.76* (0.63)
N	2721	2567	2567	2434	2434	2434
Weak IV F-stat	24.33	23.16	16.64	22.55	15.39	6.11

(b) Post-treatment periods

	g:89,t:89	g:89,t:91	g:89,t:93	g:91,t:91	g:91,t:93	g:93,t:93
(Intercept)	-1.48 (2.39)	-0.37 (3.04)	-0.68 (3.60)	2.47 (2.06)	2.90 (2.52)	-0.79 (3.46)
High School	1.10 (1.46)	1.33 (1.98)	2.56 (2.34)	0.58 (1.75)	1.62 (2.15)	1.77 (2.07)
College	3.40 (2.16)	6.20* (2.83)	7.24* (3.35)	3.64 (2.89)	4.02 (3.54)	1.27 (4.33)
Black	1.00 (1.13)	-0.69 (1.49)	-0.92 (1.77)	-2.18 (1.27)	-2.81 (1.55)	-1.55 (1.51)
White	0.78 (0.97)	-0.47 (1.30)	-0.83 (1.54)	-1.13 (1.11)	-1.66 (1.36)	-1.58 (1.32)
Female	-0.83 (1.42)	-1.39 (1.89)	-2.31 (2.24)	-1.86 (1.49)	-2.90 (1.83)	0.14 (2.78)
IFE	2.73* (0.53)	3.30* (0.68)	3.91* (0.81)	2.32* (0.58)	2.92* (0.71)	2.58* (0.93)
N	2567	2434	2434	2434	2434	2434
Weak IV F-stat	23.16	22.55	22.55	15.39	15.39	6.11

* $p < 0.05$

Notes: The table contains estimates of the interactive fixed effects model for untreated potential outcomes using AFQT as the covariate whose effect on untreated potential outcomes does not change over time and includes other covariates in the model (i.e., allowing other covariates to have time varying effects). Columns correspond to estimates for particular groups and particular time periods. For example, the first column in panel (a), which is labeled “g:89,t:87” is for the group of workers who were displaced in 1989 in the year 1987. The rows labeled “IFE” report the estimated value of F_t^* . The row labeled “Weak IV F-stat” report the F-statistic from the first stage regression of the endogenous regressors on AFQT. Some F-statistics are the same because the first stage is the same across some groups/time periods.

Table SA-4: IFE Model Estimates using Education as Covariate with Time Invariant Effect and Additional Covariates

(a) Pre-treatment periods

	g:89,t:87	g:91,t:87	g:91,t:89	g:93,t:87	g:93,t:89	g:93,t:91
(Intercept)	-2.39 (1.38)	-2.38 (1.42)	-0.42 (1.20)	-2.72 (1.58)	0.13 (1.21)	-0.97 (1.47)
Black	0.91 (0.81)	0.96 (0.83)	-0.63 (0.97)	0.75 (0.91)	-0.65 (0.98)	-1.23 (1.17)
White	0.49 (0.71)	0.42 (0.73)	-0.06 (0.86)	0.26 (0.80)	-0.18 (0.87)	-1.13 (1.04)
Female	0.82 (0.74)	0.88 (0.77)	-0.12 (0.75)	1.14 (0.87)	-0.40 (0.75)	1.27 (0.97)
IFE	2.31* (0.25)	2.30* (0.25)	2.19* (0.22)	2.39* (0.28)	2.11* (0.22)	2.13* (0.24)
N	2721	2567	2567	2434	2434	2434
Sargan p-value	0.676	0.832	0.600	0.872	0.572	0.837
Weak IV F-stat	30.37	28.99	34.24	24.62	32.87	29.49

(b) Post-treatment periods

	g:89,t:89	g:89,t:91	g:89,t:93	g:91,t:91	g:91,t:93	g:93,t:93
(Intercept)	-5.67* (2.57)	-9.83* (4.17)	-9.55* (4.74)	-0.70 (2.11)	0.94 (2.42)	0.76 (1.84)
Black	1.48 (1.51)	-0.10 (2.39)	-0.26 (2.72)	-2.62 (1.72)	-3.15 (1.96)	-1.46 (1.46)
White	0.85 (1.32)	-0.62 (2.11)	-0.92 (2.39)	-1.52 (1.51)	-1.94 (1.73)	-1.45 (1.29)
Female	1.82 (1.40)	4.27 (2.29)	3.58 (2.60)	0.43 (1.32)	-0.83 (1.51)	0.11 (1.21)
IFE	3.86* (0.45)	5.67* (0.75)	6.35* (0.85)	3.37* (0.38)	3.86* (0.44)	2.53* (0.29)
N	2567	2434	2434	2434	2434	2434
Sargan p-value	0.940	0.553	0.875	0.585	0.944	0.295
Weak IV F-stat	28.99	24.62	24.62	32.87	32.87	29.49

* $p < 0.05$

Notes: The table contains estimates of the interactive fixed effects model for untreated potential outcomes using educational attainment (this amounts to dummy variables indicating being a college graduate or being a high school graduate) as the covariates whose effects on untreated potential outcomes do not change over time and includes other covariates in the model (i.e., allowing other covariates to have time varying effects). Columns correspond to estimates for particular groups and particular time periods. For example, the first column in panel (a), which is labeled “g:89,t:87” is for the group of workers who were displaced in 1989 in the year 1987. The rows labeled “IFE” report the estimated value of F_t^* . The row labeled “Sargan p-value” provides the p-value from an over-identification test. The row labeled “Weak IV F-stat” report the F-statistic from the first stage regression of the endogenous regressors on educational attainment. Some F-statistics are the same because the first stage is the same across some groups/time periods.

Table SA-5: First Stage Regression Results by Group and Time Period

(a) Pre-Treatment Periods

	g:89,t:87	g:91,t:87	g:91,t:89	g:93,t:87	g:93,t:89	g:93,t:91
(Intercept)	1.904*	1.875*	1.376*	1.859*	1.313*	1.345*
	(0.268)	(0.280)	(0.334)	(0.292)	(0.350)	(0.415)
AFQT	0.039*	0.039*	0.044*	0.039*	0.045*	0.045*
	(0.005)	(0.005)	(0.006)	(0.005)	(0.006)	(0.007)
N	2721	2567	2567	2434	2434	2434
R^2	0.025	0.026	0.023	0.025	0.023	0.017

(b) Post-Treatment Periods

	g:89,t:89	g:89,t:91	g:89,t:93	g:91,t:91	g:91,t:93	g:93,t:93
(Intercept)	1.875*	1.859*	1.859*	1.313*	1.313*	1.345*
	(0.280)	(0.292)	(0.292)	(0.350)	(0.350)	(0.415)
AFQT	0.039*	0.039*	0.039*	0.045*	0.045*	0.045*
	(0.005)	(0.005)	(0.005)	(0.006)	(0.006)	(0.007)
N	2567	2434	2434	2434	2434	2434
R^2	0.026	0.025	0.025	0.023	0.023	0.017

Notes: This table contains results from the first stage regression of the change in earnings over time (in post treatment time periods for group g , because we allow for two years of anticipation effects, this is the change in earnings from period $g - 6$ to $g - 4$; in pre-treatment periods, it is the change in earnings from $t - 4$ to $t - 2$) among non-displaced and not-yet-displaced workers on AFQT scores. Some columns are identical because the first stage is the same across some groups/time periods. Standard errors are reported in parentheses, and a * indicates a p-value less than 0.05.

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