Supplementary Appendix: Treatment Effects in Interactive Fixed Effects Models with a Small Number of Time Periods

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This supplementary appendix contains (i) additional identification results for ATT_t when one is willing to make the additional assumption that the time varying unobservables in the model for untreated potential outcomes are serially uncorrelated or that there are strictly exogenous time varying covariates that potentially provide an additional source of moment conditions that were not available in the case considered in the main text; (ii) proofs for Theorem 2 and Propositions 1 to 4 from the main text; (iii) results from some additional Monte Carlo simulations related to weak instruments, multiple interactive fixed effects, and selecting the number of interactive fixed effects to include in the model for untreated potential outcomes; and (iv) some additional results for the application on job displacement considered in the main text.

SA-1 Alternative Approaches to Identification

In this section, we briefly consider some other possible approaches to identifying the ATT with a small number of time periods. In particular, we consider two additional approaches: (i) assuming that the time varying unobservables in the model in Assumption 1 are serially uncorrelated, and (ii) assuming access to time varying covariates in combination with a strict exogeneity condition. In principle, either approach can be used by itself or in combination with our main approach or with each other to identify the ATT.

Identification through serially uncorrelated unobservables

One common approach is to assume that the time varying unobservables, U_t in the model in Assumption 1, are mutually independent (or at least serially uncorrelated). This is a fairly common assumption in the literature on interactive fixed effects models.[1] These types of assumptions are also commonly made in the related literature on factor models with repeated measurements.

In this setup, outcomes themselves can be correlated over time, but the correlation is only allowed through the time invariant unobservables ξ and λ. Ruling out serial correlation in U_t is likely to be more plausible when one includes interactive fixed effects, but the overall plausibility of this sort of assumption is likely to be application specific, and it is one that researchers should think carefully about. For example, this kind of restriction is incompatible with the common practice of computing standard errors that are robust to serial correlation (e.g., Bertrand, Duflo, and Mullainathan (2004))

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[1]See Freyberger (2018, Section 2.1) for a recent discussion of this type of identification argument in the context of panel data models (though not treatment effect models).

in difference in differences or linear trend models. The approach outlined in this subsection requires four time periods. Three time periods are required for the "differencing" approach discussed in the main text. Another extra period of outcomes is required to use an outcome in a different time period as an instrument. We also require having access to panel data; i.e., repeated cross sections data will not be suitable for this approach. On the other hand, this approach does not require that the effect of some time invariant covariate not change over time.

Next, we introduce several more conditions required for identification of the ATT in this case.

Assumption U1 (Serially Uncorrelated Time-Varying Unobservables)**.** *The U_t are serially uncorrelated; i.e., $\mathbb{E}[U_t U_s | D = d] = 0$ for all $t \neq s$ and $d \in \{0, 1\}$.*

Assumption U1 rules out serial correlation in the time varying unobservables. Our identification argument proceeds as follows. As before, least squares estimates of (β_t^*, F_t^*) are biased because $(Y_{it^*-1} - Y_{it^*-2})$ is (by construction) correlated with V_{it}. However, under Assumption U1 and after differencing out the fixed effects as in Equation (5), Y_{is} (with $s \in \{1, 2, \dots, \mathcal{T}\} \setminus \{t, t^* - 1, t^* - 2\}$) can be used as an instrument. In particular, the following moment conditions hold:

$$\mathbb{E}[Z V_{it} | D = 0] = 0 \quad \text{and} \quad \mathbb{E}[Y_{is} V_{it} | D = 0] = 0 \text{ for } s \in \{1, 2, \dots, \mathcal{T}\} \setminus \{t, t^* - 1, t^* - 2\} \qquad \text{(SA-1)}$$

For any $t \in \{t^*, \dots, \mathcal{T}\}$, Equation (SA-1) provides $K + (\mathcal{T} - 3)$ moment equalities from the untreated group. Thus, as before, for any $t \in \{t^*, \dots, \mathcal{T}\}$, there are $K + 1$ parameters to identify, and the order condition is satisfied as long as there are at least four time periods of available data. Also, notice that this approach does not require the availability of any time invariant covariates whose effect does not change over time though if these are available, one can still incorporate these extra restrictions.

Next, we introduce a bit more notation. Let $Y_{-t,t'} = (Y_1, Y_2, \dots, Y_{t'})' \setminus \{Y_t, Y_{t^*-1}, Y_{t^*-2}\}$; i.e, the vector of outcomes across all time periods from time period 1 to t' excluding time periods t, $t^* - 1$, and $t^* - 2$. Next, define $\tilde{Z}_t = (Z', Y'_{-t,\mathcal{T}})'$ which is a $K + (\mathcal{T} - 3) \times 1$ vector of exogenous variables for period t.

Assumption U2 (Timing)**.** *$\mathcal{T} \geq 4$ and $t^* \geq 3$ (i.e., there are at least four total time periods and at least two of these are pre-treatment).*

Assumption U3 (Relevance)**.** *For $t = t^*, \dots, \mathcal{T}$, the matrix $\mathbf{M_{2t}} := \mathbb{E}\left[(1 - D)\tilde{Z}_t(Z', (Y_{t^*-1} - Y_{t^*-2}))\right]$ has full rank.*

The data requirements in Assumption U2 are strictly stronger than those required in a difference in differences type setup (typically two periods) and in models with individual-specific linear trends (typically three periods) as well as in our approach considered in the main text. Assumption U3 is a relevance condition for using outcomes in other periods as instruments. The main requirement is that $Y_{-t,\mathcal{T}}$ is correlated with $(Y_{t^*-1} - Y_{t^*-2})$ after controlling for Z. This will hold if $Y_{-t,\mathcal{T}}$ is correlated with λ after controlling for Z. This is an instrument relevance condition that can be checked with the available data.

Proposition SA-1. *Under Assumptions 1, 2, 4, 5, U1, U2, and U3, β_t^* and F_t^* are identified for all $t = t^*, \dots, \mathcal{T}$. In particular, for any $m_2 \times m_2$ (with $m_2 := K + \mathcal{T} - 3$) positive definite weighting matrix $\mathbf{W}$,*

$$(\beta_t^{*\prime}, F_t^*)' = (\mathbf{M'_{2t}} \mathbf{W} \mathbf{M_{2t}})^{-1} \mathbf{M'_{2t}} \mathbf{W} \mathbb{E}[(1 - D)\tilde{Z}_t(Y_t - Y_{t^*-2})]$$

In addition, ATT_t is identified for all $t = t^*, \dots, \mathcal{T}$, and it is given by

$$ATT_t = \mathbb{E}[Y_t - Y_{t^*-2} | D = 1] - \Big(\mathbb{E}[X' | D = 1]\beta_t^* + F_t^*(\mathbb{E}[Y_{t^*-1} - Y_{t^*-2} | D = 1])\Big)$$

Proof. The result holds immediately using the same arguments as in the proof of Theorem 1. □

Remark SA-1. *There are alternative approaches that could be considered under the same assumptions. For example, one could use the covariance matrix of outcomes over time for the untreated group and pre-treatment outcomes for the treated group to identify the parameters of the model. See Williams (2020) as a recent, relevant example in a different context though noting that similar arguments are likely to apply in our setting as well.*

Finally, it is useful to compare our results in this section to existing results for factor models that use similar independence conditions to obtain identification. The setup of the model in Assumption 1 combined with assumptions in this subsection is quite similar to a factor model with two factors. A typical setup there would require independence of the unobservables across equations (corresponding to our condition that the time varying unobservables be uncorrelated with each other as in Assumption U1). That case typically requires five "measurements" (corresponding to our number of time periods). The number of time periods in our case is reduced by one because the coefficient on one of the time invariant unobservables, ξ, is restricted not to change over time. Thus, these results are quite similar. More interestingly though, in our case, even when there are only four time periods, either ATT_3 is identified (this happens if $t^* = 3$) or there are testable implications of our approach (this happens if $t^* = 4$). The reason for this is that we can identify $(\beta_3^*, F_3^*, \beta_4^*, F_4^*)$ only using information from the untreated group.

Identification through time varying covariates

To extend the model in Assumption 1 to the case with time-varying covariates, consider the following model for untreated potential outcomes

$$Y_{it}(0) = \theta_t + \xi_i + \lambda_i F_t + X_{it}'\beta + U_{it} \tag{SA-2}$$

In principle, it is straightforward to accommodate both time invariant and time varying covariates simultaneously, but, for simplicity, we consider the case where there are only time varying covariates here. Following essentially the same differencing arguments as in the main text, we can write

$$Y_{it}(0) - Y_{it^*-2} = \theta_t^* + (X_{it} - X_{it^*-2})'\beta + F_t^*(Y_{it^*-1} - Y_{it^*-2}) - (X_{it^*-1} - X_{it^*-2})'\beta F_t^* + V_{it}$$

where $V_{it} = (U_{it} - U_{it^*-2}) - F_t^*(U_{it^*-1} - U_{it^*-2})$. Consider the following assumption.

Assumption X1 (Strict Exogeneity)**.** *Let $X^{\mathcal{T}} = (1, X_1', \ldots, X_{\mathcal{T}}')'$. For all $(t, d) \in \{1, \ldots, \mathcal{T}\} \times \{0, 1\}$, $\mathbb{E}[U_t | X^{\mathcal{T}}, \xi, \lambda, D = d] = 0$.*

Assumption X1 is a common strict exogeneity assumption for time varying covariates in the context of panel data models. It implies that U_t (the time varying unobservables) are uncorrelated with the covariates in all time periods. It seems possible to extend the arguments in this section to alternative assumptions on the covariates such as pre-determinedness (though this might require adjusting the differencing strategy that we have been using throughout the paper); see, for example, Arellano and Honoré (2001) for discussion of these types of assumptions. Assumption X1 allows us to use covariates in periods besides period t as instruments for $(Y_{t^*-1} - Y_{t^*-2})$. Finally, we make assumptions about the sampling scheme, treatment timing, and a relevance assumption.

Assumption X2 (Observed Data)**.** *The observed data consists of iid draws of $\{Y_{i1}, Y_{i2}, \ldots, Y_{i\mathcal{T}}, X_{i1}, X_{i2}, \ldots, X_{i\mathcal{T}}, D_i\}_{i=1}^n$, where n denotes the sample size.*

Assumption X3 (Timing)**.** $\mathcal{T} \geq 3$ *and* $t^* \geq 3$ *(i.e., there are at least three total time periods and at least two pre-treatment time periods).*

Assumption X4 (Relevance). *For $t = t^*, \ldots, \mathcal{T}$, the matrix $\mathbf{M_{X_t}} := \mathbb{E}[(1-D)X^{\mathcal{T}}(1, (X_t - X_{t^*-2})', Y_{t^*-1} - Y_{t^*-2}, (X_{t^*-1} - X_{t^*-2})')]$ has full rank.*

Assumption X2 extends the iid panel sampling scheme to allow for time varying covariates. Assumption X3 is the same as Assumption 3 in the main text. Assumption X4 is an instrument relevance condition for using covariates in alternative time periods as an instrument to identify F_t^*.

Proposition SA-2. *In the model in Equation (SA-2) and under Assumptions 4, X1, X2, X3, and X4, β_t^* and F_t^* are identified for all $t = t^*, \ldots, \mathcal{T}$. In particular, for any $m_3 \times m_3$ (with m_3 the number of rows of $\mathbf{M_{X_t}}$) positive definite weighting matrix $\mathbf{W}$,*

$$(\theta_t^*, \beta', F_t^*, \zeta_t')' = (\mathbf{M'_{Xt}} \mathbf{W} \mathbf{M_{Xt}})^{-1} \mathbf{M'_{Xt}} \mathbf{W} \mathbb{E}[(1-D)X^{\mathcal{T}}(Y_t - Y_{t^*-2})]$$

where ζ_t is an extra nuisance parameter that is equal to βF_t^* (though we consider it separately here). In addition, ATT_t is identified for all $t = t^*, \ldots, \mathcal{T}$, and it is given by

$$ATT_t = \mathbb{E}[Y_t - Y_{t^*-2}|D=1] - \Big(\theta_t^* + \mathbb{E}[(X_t - X_{t^*-2})'|D=1]\beta + F_t^*(\mathbb{E}[Y_{t^*-1} - Y_{t^*-2}|D=1]) - \mathbb{E}[(X_{t^*-1} - X_{t^*-2})'|D=1]\zeta_t\Big)$$

Proof. The result holds immediately using the same arguments as in the proof of Theorem 1. □

To conclude this section, it is worth making a few comments about the sort of model in Equation (SA-2). First (and momentarily ignoring the interactive fixed effect term), there are some important conceptual issues with this sort of model in the context of policy evaluation. In the presence of time varying covariates, work in the econometrics literature on difference in differences where the parallel trends assumption holds after conditioning on covariates (e.g., Heckman, Ichimura, Smith, and Todd (1998), Abadie (2005), and Callaway and Sant'Anna (2021)) typically conditions on a particular pre-treatment value of the covariates. This sort of setup uses the group of individuals who do not participate in the treatment and have the same value of the pre-treatment covariates as the comparison group. This is consistent with the approach in the current paper of including a pre-treatment value of a covariate that varies over time but allowing its effect to change over time. On the other hand, the model in Equation (SA-2) effectively uses individuals whose covariates *change* by the same amount over time (though whose level could be quite different) as the comparison group. This does not seem to be the interpretation that most applied work is aiming for. A second main issue with the approach in this section is that, even if the model in Equation (SA-2) is correct (and hence β and F_t^* can be identified), covariates that can vary over time may be affected by participating in the treatment. In this case, it would probably make sense to define treated and untreated potential covariates (these sorts of points are made in Lechner (2008) and Bonhomme and Sauder (2011)). Then, identifying the ATT would require identifying the average of untreated potential covariates for the treated group. However, this term is not immediately identified and would likely require a number of additional assumptions and perhaps introduce other complications as well.

SA-2 Additional Proofs

In this section, we provide the proofs for Theorem 2 and Propositions 1 to 4 from the main text.

Proof of Theorem 2

To start with, we prove an intermediate result providing an asymptotically linear representation for the first stage estimates coming from the interactive fixed effects model for untreated potential outcomes.

Lemma SA-2.1. *Under Assumptions 1 to 8 and for any* $t \in \{t^*, \ldots, \mathcal{T}\}$,

$$\sqrt{n}\begin{pmatrix} \hat{\beta}_t^* - \beta_t^* \\ \hat{F}_t^* - F_t^* \end{pmatrix} = \frac{1}{\sqrt{n}}\sum_{i=1}^n \varphi_t^{GMM}(D_i, Z_i, Y_i) + o_p(1)$$

Proof. Start by defining

$$\varphi_\gamma(D_i, Z_i, Y_i) := (\mathbb{E}[\mathbf{ZX}]'\mathbf{W}\mathbb{E}[\mathbf{ZX}])^{-1}\mathbb{E}[\mathbf{ZX}]'\mathbf{W}\mathbf{Z}_i V_i$$

Standard arguments on GMM immediately imply that

$$\sqrt{n}(\hat{\gamma} - \gamma) = \frac{1}{\sqrt{n}}\sum_{i=1}^n \psi_\gamma(D_i, Z_i, Y_i) + o_p(1) \tag{SA-3}$$

To complete the proof, notice that

$$\begin{aligned}
\sqrt{n}\begin{pmatrix} \hat{\beta}_t^* - \beta_t^* \\ \hat{F}_t^* - F_t^* \end{pmatrix} &= \mathbf{\Xi}_t\sqrt{n}(\hat{\gamma} - \gamma) \\
&= \mathbf{\Xi}_t\frac{1}{\sqrt{n}}\sum_{i=1}^n \psi_\gamma(D_i, Z_i, Y_i) + o_p(1) \\
&= \frac{1}{\sqrt{n}}\sum_{i=1}^n \varphi_t^{GMM}(D_i, Z_i, Y_i) + o_p(1)
\end{aligned}$$

where the first equality holds immediately, the second equality holds by Equation (SA-3), and the last equality holds by the definition of φ_t^{GMM}. □

Proof of Theorem 2. First, notice that

$$\sqrt{n}(\hat{p} - p) = \frac{1}{\sqrt{n}}\sum_{i=1}^n \varphi^P(D_i)$$

Next, notice that

$$\begin{aligned}
&\sqrt{n}\left(\widehat{ATT}_t - ATT_t\right) \\
&\quad = \sqrt{n}\left(\hat{p}^{-1} - p^{-1}\right)\left(\frac{1}{n}\sum_{i=1}^n D_i(Y_{it} - Y_{it^*-2}) - \frac{1}{n}\sum_{i=1}^n D_i X_i' \hat{\beta}_t^* - \hat{F}_t^* \frac{1}{n}\sum_{i=1}^n D_i(Y_{it^*-1} - Y_{it^*-2})\right) \\
&\qquad + p^{-1}\frac{1}{\sqrt{n}}\sum_{i=1}^n \varphi_t^M(D_i, X_i, Y_i)' \begin{pmatrix} 1 \\ \hat{\beta}_t^* \\ \hat{F}_t^* \end{pmatrix} \\
&\qquad + p^{-1}\left[\begin{pmatrix} \mathbb{E}[DX] \\ \mathbb{E}[D(Y_{t^*-1} - Y_{t^*-2})] \end{pmatrix}' \sqrt{n}\begin{pmatrix} \hat{\beta}_t^* - \beta_t^* \\ \hat{F}_t^* - F_t^* \end{pmatrix}\right] \\
&\quad := A + B + C
\end{aligned}$$

which follows by the expressions for $\widehat{ATT}_t$ and ATT_t in Equations (16) and (17), by adding and subtracting several terms, and by the definition of φ_t^M. We consider A, B, and C in turn.

First, consider term A.

$$\begin{aligned}
A &= -\frac{\sqrt{n}(\hat{p} - p)}{\hat{p}p}\left(\frac{1}{n}\sum_{i=1}^n D_i(Y_{it} - Y_{it^*-2}) - \frac{1}{n}\sum_{i=1}^n D_i X_i' \hat{\beta}_t^* - \hat{F}_t^* \frac{1}{n}\sum_{i=1}^n D_i(Y_{it^*-1} - Y_{it^*-2})\right) \\
&= -\frac{ATT_t}{p^2}\frac{1}{\sqrt{n}}\sum_{i=1}^n \varphi^P(D_i) + o_p(1) \\
&= \frac{1}{\sqrt{n}}\sum_{i=1}^n \psi_t^A(D_i) + o_p(1)
\end{aligned}$$

where the first equality holds by the definition of term A and by multiplying and dividing the first term by p and the second term by $\hat{p}$ and then combining terms, the second equality holds by the weak law of large numbers and continuous mapping theorem, and the last equality holds by the definition of ψ_t^A.

Next, consider term B.

$$B = \frac{1}{\sqrt{n}}\sum_{i=1}^n \psi_t^B(D_i, X_i, Y_i) + o_p(1)$$

which immediately holds from the definition of term B because $\hat{\beta}_t^*$ and $\hat{F}_t^*$ are consistent for β_t^* and F_t^* and by the definition of ψ_t^B.

Finally, consider term C. From Lemma SA-2.1 and the continuous mapping theorem, it immediately holds that

$$\begin{aligned}
C &= p^{-1}\begin{pmatrix} \mathbb{E}[DX] \\ \mathbb{E}[D(Y_{t^*-1} - Y_{t^*-2})] \end{pmatrix}' \frac{1}{\sqrt{n}}\sum_{i=1}^n \varphi_t^{GMM}(D_i, Z_i, Y_i) + o_p(1) \\
&= \frac{1}{\sqrt{n}}\sum_{i=1}^n \psi_t^C(D_i, Z_i, Y_i) + o_p(1)
\end{aligned}$$

Combining the expressions for terms A, B, and C implies that, for any $t \in \{t^*, \dots, \mathcal{T}\}$

$$\sqrt{n}\left(\widehat{ATT}_t - ATT_t\right) = \frac{1}{\sqrt{n}}\sum_{i=1}^n \psi_t(D_i, Z_i, Y_i) + o_p(1) \tag{SA-4}$$

which holds by the definition of ψ_t and completes the first part of the proof. The second part of the proof holds by applying the central limit theorem jointly to the above expression for all $t = t^*, \ldots, \mathcal{T}$. □

Proofs of Propositions 1 to 4

The results in Propositions 1, 3, and 4 all hold using essentially the same arguments as in Theorem 1. For Proposition 2, the proof is also similar but we need to account for the repeated cross sections sampling scheme in Assumption RC.

Proof of Proposition 2. First, the validity of the moment conditions in Equation (22) holds immediately from Assumption RC. This implies the result in the first part of the proposition concerning identification of γ (the parameters from the interactive fixed effects model for untreated potential outcomes). For the second part of the result, notice that

$$\begin{aligned}
ATT_t &= \mathbb{E}[Y_t - Y_{t^*-2}|D=1] - \Big(\mathbb{E}[X'|D=1]\beta_t^* + F_t^*\mathbb{E}[Y_{t^*-1} - Y_{t^*-2}|D=1]\Big) \\
&= \big(\mathbb{E}_M[Y|D=1, T=t] - \mathbb{E}_M[Y|D=1, T=t^*-2]\big) \\
&\qquad - \Big(\mathbb{E}_M[X'|D=1]\beta_t^* + F_t^*\Big(\mathbb{E}_M[Y|D=1, T=t^*-1] - \mathbb{E}_M[Y|D=1, T=t^*-2]\Big) \\
&= \mathbb{E}_M\left[\frac{T_t}{\pi_t}Y - \frac{T_{t^*-2}}{\pi_{t^*-2}}Y \Big| D=1\right] \\
&\qquad\qquad - \left\{\mathbb{E}_M[X'|D=1]\beta_t^* + F_t^*\left(\mathbb{E}_M\left[\frac{T_{t^*-1}}{\pi_{t^*-1}}Y - \frac{T_{t^*-2}}{\pi_{t^*-2}}Y \Big| D=1\right]\right)\right\}
\end{aligned}$$

where the first equality holds from the same arguments as in Theorem 1, and second and third equalities hold by Assumption RC. □

SA-3 Additional Monte Carlo Simulations

This section contains additional Monte Carlo simulations related to weak instruments, estimation with multiple interactive fixed effects, and selecting the number of interactive fixed effects in the model for untreated potential outcomes.

Multiple IFEs and Varying Instrument Strength

Relative to the main text, our first set of simulation results in this section allow for multiple interactive fixed effects and vary the strength of the instrument. We consider the same setup as in the main text except that we modify the model for untreated potential outcomes to be

$$Y_{it}(0) = \theta_t + \xi_i + \lambda_i' F_t + W_i'\alpha + U_{it}$$

where λ and W both contain three elements. For $j = 1, 2, 3$, $W_{ij} \sim N(0, 1)$ and is independent of all other random variables; $\alpha_1 = 1$, $\alpha_2 = -1$, and $\alpha_3 = 0$; $\lambda_{i1} = 1 + 2D_i + \rho_1 W_{i1} + \epsilon_{i1}$, $\lambda_{i2} = 1 - 5D_i + \rho_2 W_{i2} + \epsilon_{i2}$, $\lambda_{i3} = 5 - 10D_i + \rho_3 W_{i3} + \epsilon_{i3}$ where $\epsilon_{ij} \sim N(0, 0.1)$ and is independent of all other random variables. For all simulations, we set $\rho_1 = \rho_2 = \rho_3 = \rho$ and vary ρ across simulations. Next, F_t, which we allow to vary across designs, also contains three elements. First, we set $\tilde{F}_{1t} = t$, $\tilde{F}_{2t} = (-1)^t t \log(t)$ (i.e., $\tilde{F}_{2t}$ alternates between being positive and negative but is increasing in magnitude), and $\tilde{F}_{3t} = (-1)^{\mathbf{1}\{t \geq 4\}}(3 - |3 - t|)^2$ (i.e., $(\tilde{F}_{31}, \tilde{F}_{32}, \tilde{F}_{33}, \tilde{F}_{34}, \tilde{F}_{35}) = (1^2, 2^2, 3^2, -2^2, -1^2)$). In designs where the true model for untreated potential outcomes includes 0 interactive fixed effects, we set $F_t = (0, 0, 0)$ in all time periods; when the true model includes 1 interactive fixed effect, we set $F_t = (\tilde{F}_{1t}, 0, 0)$; when the true model includes 2 or 3 interactive fixed effects, we set $F_t = (\tilde{F}_{1t}, \tilde{F}_{2t}, 0)$ or $F_t = (\tilde{F}_{1t}, \tilde{F}_{2t}, \tilde{F}_{3t})$, respectively.

These results are provided in Table SA-1. First, depending on the model, the fraction of models that pass the weak IV diagnostics (just that all F-statistics from the first stage are greater than 10) range from 0.66 to 1 for the case when $\rho = 0.2$, when $\rho = 1$ the model always passes the weak IV diagnostics, and when $\rho = 0.05$, the model always fails weak IV diagnostics.

Next, we discuss the top panel of the table where W_i is a strong instrument. In the case where the estimated model includes the correct number of interactive effects, our estimator performs well across designs. Bias and root mean squared error are generally small, and our approach rejects at close to the nominal level of the test across all cases. When the estimated model includes an incorrect number of interactive fixed effects, one very interesting pattern from that panel is that the consequences of including the wrong number of interactive fixed effects appears to be very asymmetric. In particular, including too few interactive fixed effects generally appears to lead to severely biased estimates and large root mean squared errors. This is in stark contrast to including too many interactive fixed effects where the consequences are much less severe — slightly larger bias, slightly larger root mean squared error, and the tendency to under-reject. This pattern is broadly similar to results in Moon and Weidner (2015).

The second panel considers the case where W_i is a moderately strong instrument for λ_i. When the estimated model includes the correct number of interactive fixed effects, the bias and root mean squared errors are somewhat larger in this case than in the strong instrument case. Furthermore, as expected, our inference procedure performs somewhat worse in this case — the rejection probabilities tend to be somewhat further away from the nominal level of the test. Like the previous case, the consequences of including too few interactive fixed effects appear more severe than the consequences of including too many interactive fixed effects. When we move to the case with a weak instrument in the third panel, the results are notably worse; even in the case where the number of interactive fixed effects is correctly specified, bias and root mean squared error are notably higher. Our inference

Table SA-1: Monte Carlo Simulations Varying Instrument Strength and with Multiple IFEs

	Truth															
	0 IFE				1 IFE				2 IFE				3 IFE			
	Bias	RMSE	MAD	Rej.	Bias	RMSE	MAD	Rej.	Bias	RMSE	MAD	Rej.	Bias	RMSE	MAD	Rej.
$\rho = 1$																
$IFE = 0$	0.000	0.039	0.027	0.050	2.004	2.006	2.002	1.000	69.951	69.963	69.929	1.000	39.929	39.952	39.951	1.000
$IFE = 1$	0.002	0.049	0.028	0.025	0.007	0.121	0.085	0.053	6.424	6.488	6.440	1.000	95.868	95.948	95.844	1.000
$IFE = 2$	0.000	0.118	0.036	0.012	-0.002	0.197	0.078	0.010	-0.007	0.343	0.231	0.055	45.917	46.140	45.918	1.000
$IFE = 3$	1.190	38.451	0.055	0.000	-0.023	2.513	0.118	0.001	-0.108	5.955	0.305	0.011	-0.041	1.536	1.081	0.065
$\rho = 0.2$																
$IFE = 0$	0.000	0.040	0.026	0.044	1.999	2.000	1.997	1.000	69.957	69.959	69.950	1.000	39.969	39.972	39.951	1.000
$IFE = 1$	0.004	0.049	0.028	0.021	0.071	0.509	0.349	0.084	6.436	6.763	6.389	0.925	95.857	96.247	95.595	1.000
$IFE = 2$	-0.003	0.093	0.035	0.007	0.032	0.723	0.322	0.025	-0.066	1.662	1.093	0.040	46.109	47.517	46.461	0.978
$IFE = 3$	-0.091	1.167	0.054	0.002	0.572	11.313	0.492	0.003	1.480	38.273	1.603	0.010	-0.500	7.997	5.000	0.062
$\rho = 0.05$																
$IFE = 0$	0.000	0.041	0.028	0.052	2.001	2.002	2.000	1.000	69.957	69.958	69.954	1.000	39.960	39.962	39.957	1.000
$IFE = 1$	-0.002	0.048	0.028	0.020	1.115	1.944	1.281	0.246	7.057	12.884	7.663	0.297	97.206	102.913	95.756	0.898
$IFE = 2$	0.001	0.070	0.032	0.007	0.488	2.879	1.153	0.027	0.582	11.586	4.492	0.026	51.836	122.970	55.015	0.341
$IFE = 3$	-0.007	0.788	0.053	0.000	2.483	53.180	1.560	0.003	-3.797	210.037	6.434	0.001	-36.496	1088.472	26.704	0.016

Notes: Each group of columns in the table corresponds to the true number of interactive fixed effects in the model for untreated potential outcomes. Columns labeled "Bias" report the simulated bias, columns labeled "RMSE" report root mean squared error, columns labeled "MAD" report median absolute deviation, and columns labeled "Rej." report the reject probabilities for a test of the null hypothesis that $ATT_5 = 0$ (which is true here) at the 5% significance level. The top panel, where $\rho = 1$, reports results with a strong instrument; the middle panel provides results with a "medium" strength instrument, and the bottom panel reports results with a weak instrument. Rows labeled "$IFE = j$" correspond to estimates coming from a model that includes j interactive fixed effects. Each simulation uses $n = 1000$. Results come from 1000 Monte Carlo simulations.

procedure performs poorly in this case too — in these simulations, it over-rejects when the true number of interactive fixed effects is one and under-rejects when the true number of interactive fixed effects is two or three. That being said, we stress that researchers would typically be able to avoid this case by using standard weak IV diagnostics.

Model Selection

To conclude our Monte Carlo simulations, we provide simulations related to selecting the number of interactive fixed effects in the model for untreated potential outcomes as we discussed in Section 5.1 in the main text. We use the same DGPs as in the previous section.

Our results on model selection are provided in Table SA-2 (for the case with a strong instrument; i.e., $\rho = 1$) and in Table SA-3 (for the case with a "medium strength" instrument; i.e., $\rho = 0.2$). In the case with a strong instrument, BIC seems to perform best among the three model selection criteria (at least in our particular simulations) — generally, BIC chooses the correct model with a high probability even in cases where the number of observations is small. In the extreme cases where the true number of interactive fixed effects is equal to 0 or 3, BIC chooses the correct model 100% of the time; in the middle cases, where the true number of interactive fixed effects is 1 or 2, BIC chooses the correct model at least 96% of the time across different numbers of observations. The two types of cross validation approaches appear to perform roughly equally. They both choose the correct number of interactive fixed effects less often than BIC, and they are generally conservative (in the sense of, when they choose an incorrect model, they tend to choose a model that includes too many interactive fixed effects rather than one that includes too few interactive fixed effects).

Table SA-2: Model Selection with Strong Instrument

	BIC(0)	BIC(1)	BIC(2)	BIC(3)	CV-U(0)	CV-U(1)	CV-U(2)	CV-U(3)	CV-T(0)	CV-T(1)	CV-T(2)	CV-T(3)
$IFE = 0$												
$n = 100$	1	0	0	0	0.458	0.428	0.104	0.010	0.376	0.400	0.185	0.039
$n = 500$	1	0	0	0	0.370	0.441	0.162	0.027	0.333	0.414	0.205	0.048
$n = 1000$	1	0	0	0	0.345	0.440	0.186	0.029	0.325	0.438	0.194	0.043
$IFE = 1$												
$n = 100$	0	0.976	0.024	0	0	0.416	0.480	0.104	0	0.309	0.472	0.219
$n = 500$	0	0.988	0.012	0	0	0.303	0.509	0.188	0	0.267	0.502	0.231
$n = 1000$	0	0.993	0.007	0	0	0.317	0.497	0.186	0	0.287	0.488	0.225
$IFE = 2$												
$n = 100$	0	0	0.968	0.032	0	0	0.785	0.215	0	0	0.680	0.320
$n = 500$	0	0	0.985	0.015	0	0	0.672	0.328	0	0	0.649	0.351
$n = 1000$	0	0	0.989	0.011	0	0	0.709	0.291	0	0	0.677	0.323
$IFE = 3$												
$n = 100$	0	0	0	1	0	0	0	1	0	0	0	1
$n = 500$	0	0	0	1	0	0	0	1	0	0	0	1
$n = 1000$	0	0	0	1	0	0	0	1	0	0	0	1

Notes: The table provides estimates of the fraction of times that different model selection criteria (BIC, cross validation using the untreated group, and cross validation using the treated group) chose different numbers of interactive fixed effects in the model for untreated potential outcomes. Results are computed using 1000 Monte Carlo simulations and vary the true number of interactive fixed effects and number of observations across simulations. Panels in the table are separated by the true number of interactive fixed effects in the model for untreated potential outcomes (e.g., in the top panel, labeled "$IFE = 0$", the true number of interactive fixed effects is 0). Within each panel, the number of observations varies among 100, 500, and 1000. Columns labeled "BIC(j)" provide the fraction of times that model j was chosen using the BIC model selection criteria. Columns labeled "CV-U(j)" provide the fraction of times that model j was chosen using cross validation among the untreated group (as discussed in the text). Columns labeled "CV-T(j)" provide the fraction of times that model j was chosen using cross validation among the treated group (as discussed in the text). Thus, for example, the value 0.458 in the first row and column labeled "CV-U(0)" indicates that cross validation using the untreated group selected the model with zero interactive fixed effects in 458 out of 1000 simulations when the true number of interactive fixed effects was zero and there were 100 observations.

Next, we consider the results in Table SA-3 with a medium strength instrument. In general, BIC continues to perform best in these simulations though cross-validation using the treated group appears to perform somewhat better than cross-validation using the untreated group. Although the results are broadly similar to the ones above in the case with a stronger instrument, there is one difference that is worth highlighting. In the case with one interactive fixed effect and $n = 100$, BIC selects the model with 0 interactive fixed effects 92% of the time; in the case with $n = 500$, BIC still selects this model 11% of the time. On the other hand, cross-validation using the treated group selects the model with 0 interactive fixed effects 3% of the time (when $n = 100$) and 0% of the time (when $n = 500$). Given the results in the previous section where our estimators performed substantially worse when one includes too few interactive fixed effects relative to the case when one includes too many interactive fixed effects, it suggests that, at least in this case, choosing the model based on cross validation using the treated group is likely to work better than choosing the model based on BIC. This suggests both BIC and our cross-validation approaches (particularly, cross-validation using the treated group) are worth consulting for choosing the number of interactive fixed effects to include in the model for untreated potential outcomes.

Table SA-3: Model Selection with "Medium" Strength Instrument

	BIC(0)	BIC(1)	BIC(2)	BIC(3)	CV-U(0)	CV-U(1)	CV-U(2)	CV-U(3)	CV-T(0)	CV-T(1)	CV-T(2)	CV-T(3)
$IFE = 0$												
$n = 100$	1	0	0	0	0.459	0.430	0.097	0.014	0.360	0.421	0.168	0.051
$n = 500$	1	0	0	0	0.363	0.450	0.167	0.020	0.335	0.433	0.187	0.045
$n = 1000$	1	0	0	0	0.334	0.439	0.188	0.039	0.313	0.434	0.199	0.054
$IFE = 1$												
$n = 100$	0.924	0.075	0.001	0	0.828	0.033	0.104	0.035	0.027	0.369	0.396	0.208
$n = 500$	0.108	0.885	0.007	0	0.772	0	0.115	0.113	0	0.298	0.463	0.239
$n = 1000$	0	0.997	0.003	0	0.799	0	0.091	0.110	0	0.295	0.479	0.226
$IFE = 2$												
$n = 100$	0	0.096	0.868	0.036	0	0.210	0.643	0.147	0	0.142	0.579	0.279
$n = 500$	0	0	0.981	0.019	0	0.033	0.672	0.295	0	0.006	0.676	0.318
$n = 1000$	0	0	0.994	0.006	0	0.006	0.683	0.311	0	0	0.703	0.297
$IFE = 3$												
$n = 100$	0	0	0.021	0.979	0.001	0.026	0.086	0.887	0.069	0	0.080	0.851
$n = 500$	0	0	0	1	0	0	0	1	0	0	0.003	0.997
$n = 1000$	0	0	0	1	0	0	0	1	0	0	0	1

Notes: The table provides estimates of the fraction of times that different model selection criteria (BIC, cross validation using the untreated group, and cross validation using the treated group) chose different numbers of interactive fixed effects in the model for untreated potential outcomes. Results are computed using 1000 Monte Carlo simulations and vary the true number of interactive fixed effects and number of observations across simulations. Panels in the table are separated by the true number of interactive fixed effects in the model for untreated potential outcomes (e.g., in the top panel, labeled "$IFE = 0$", the true number of interactive fixed effects is 0). Within each panel, the number of observations varies among 100, 500, and 1000. Columns labeled "BIC(j)" provide the fraction of times that model j was chosen using the BIC model selection criteria. Columns labeled "CV-U(j)" provide the fraction of times that model j was chosen using cross validation among the untreated group (as discussed in the text). Columns labeled "CV-T(j)" provide the fraction of times that model j was chosen using cross validation among the treated group (as discussed in the text). Thus, for example, the value 0.459 in the first row and column labeled "CV-U(0)" indicates that cross validation using the untreated group selected the model with zero interactive fixed effects in 459 out of 1000 simulations when the true number of interactive fixed effects was zero and there were 100 observations.

In Table SA-4, we provide analogous simulation results for the case with a weak instrument (i.e., $\rho = 0.05$). All three model selection procedures perform worse in this case though we again emphasize that the researcher would generally be able to detect that they are in the case with a weak instrument (using well-known diagnostics related to weak instruments) and would, therefore, be unlikely to rely heavily on this model selection procedure.

Table SA-4: Model Selection with Weak Instrument

	BIC(0)	BIC(1)	BIC(2)	BIC(3)	CV-U(0)	CV-U(1)	CV-U(2)	CV-U(3)	CV-T(0)	CV-T(1)	CV-T(2)	CV-T(3)
$IFE = 0$												
$n = 100$	1	0	0	0	0.476	0.418	0.097	0.009	0.365	0.392	0.195	0.048
$n = 500$	1	0	0	0	0.360	0.399	0.214	0.027	0.342	0.393	0.217	0.048
$n = 1000$	1	0	0	0	0.354	0.438	0.174	0.034	0.336	0.426	0.197	0.041
$IFE = 1$												
$n = 100$	0.999	0.001	0	0	0.926	0.033	0.029	0.012	0.237	0.275	0.303	0.185
$n = 500$	0.999	0.001	0	0	0.882	0.013	0.071	0.034	0.120	0.336	0.352	0.192
$n = 1000$	0.995	0.005	0	0	0.852	0.003	0.077	0.068	0.059	0.367	0.369	0.205
$IFE = 2$												
$n = 100$	0.029	0.827	0.141	0.003	0.013	0.523	0.398	0.066	0.001	0.330	0.396	0.273
$n = 500$	0.007	0.677	0.312	0.004	0	0.315	0.506	0.179	0	0.231	0.490	0.279
$n = 1000$	0	0.425	0.568	0.007	0	0.265	0.518	0.217	0	0.183	0.526	0.291
$IFE = 3$												
$n = 100$	0.040	0.144	0.319	0.497	0.141	0.361	0.275	0.223	0.451	0.039	0.170	0.340
$n = 500$	0.009	0.038	0.210	0.743	0.019	0.192	0.180	0.609	0.249	0.012	0.148	0.591
$n = 1000$	0.002	0.009	0.069	0.920	0.004	0.074	0.125	0.797	0.125	0.004	0.107	0.764

Notes: The table provides estimates of the fraction of times that different model selection criteria (BIC, cross validation using the untreated group, and cross validation using the treated group) chose different numbers of interactive fixed effects in the model for untreated potential outcomes. Results are computed using 1000 Monte Carlo simulations and vary the true number of interactive fixed effects and number of observations across simulations. Panels in the table are separated by the true number of interactive fixed effects in the model for untreated potential outcomes (e.g., in the top panel, labeled "$IFE = 0$", the true number of interactive fixed effects is 0). Within each panel, the number of observations varies among 100, 500, and 1000. Columns labeled "BIC(j)" provide the fraction of times that model j was chosen using the BIC model selection criteria. Columns labeled "CV-U(j)" provide the fraction of times that model j was chosen using cross validation among the untreated group (as discussed in the text). Columns labeled "CV-T(j)" provide the fraction of times that model j was chosen using cross validation among the treated group (as discussed in the text). Thus, for example, the value 0.476 in the first row and column labeled "CV-U(0)" indicates that cross validation using the untreated group selected the model with zero interactive fixed effects in 476 out of 1000 simulations when the true number of interactive fixed effects was zero and there were 100 observations.

SA-4 Additional Results on Job Displacement

This section contains some additional results for the application in the main text about job displacement. In particular, it provides estimates for the interactive fixed effects model for untreated potential outcomes when (i) education is used as the covariate with time invariant effects, (ii) AFQT is used as the covariate with time invariant effects and additional covariates are included in the model, and (iii) education is used as the covariate with time invariant effects and additional covariates are included in the model. In addition, it contains the complete first stage estimates for the main case considered in the main text where AFQT is used as the covariate with time invariant effects and no additional covariates are included.

Table SA-5: IFE Model Estimates using Education as Covariate with Time Invariant Effect

(a) Pre-Treatment Periods

	g:89,t:87	g:91,t:87	g:91,t:89	g:93,t:87	g:93,t:89	g:93,t:91
(Intercept)	−1.48	−1.46	−0.73	−1.84	−0.38	−1.14
	(0.99)	(1.02)	(0.86)	(1.12)	(0.84)	(0.92)
IFE	2.32*	2.31*	2.21*	2.42*	2.12*	2.10*
	(0.25)	(0.25)	(0.22)	(0.28)	(0.22)	(0.23)
N	2721	2567	2567	2588	2434	2434
Sargan p-value	0.527	0.677	0.589	0.784	0.528	0.802
Weak IV F-stat	29.09	27.59	33.65	23.45	32.54	29.58

* $p < 0.05$

(b) Post-Treatment Periods

	g:89,t:89	g:89,t:91	g:89,t:93	g:91,t:91	g:91,t:93	g:93,t:93
(Intercept)	−4.00*	−8.10*	−8.17*	−1.95	−1.23	−0.17
	(1.86)	(3.05)	(3.42)	(1.47)	(1.66)	(1.13)
IFE	3.91*	5.73*	6.35*	3.36*	3.80*	2.47*
	(0.46)	(0.76)	(0.86)	(0.38)	(0.43)	(0.28)
N	2567	2434	2434	2434	2434	2434
Sargan p-value	0.869	0.719	0.995	0.554	0.932	0.341
Weak IV F-stat	27.59	23.45	23.45	32.54	32.54	29.58

* $p < 0.05$

Notes: The table contains estimates of the interactive fixed effects model for untreated potential outcomes using educational attainment (this amounts to dummy variables indicating being a college graduate or being a high school graduate) as the covariates whose effects on untreated potential outcomes do not change over time and without including other covariates. Columns correspond to estimates for particular groups and particular time periods. For example, the first column in panel (a), which is labeled "g:89,t:87" is for the group of workers who were displaced in 1989 in the year 1987. The rows labeled "IFE" report the estimated value of $F^*_{g,t}$. The row labeled "Sargan p-value" provides the p-value from an over-identification test. The row labeled "Weak IV F-stat" report the F-statistic from the first stage regression of the endogenous regressors on educational attainment. Some F-statistics are the same because the first stage is the same across some groups/time periods.

Table SA-6: IFE Model Estimates using AFQT as Covariate with Time Invariant Effect and Additional Covariates

(a) Pre-treatment periods

	g:89,t:87	g:91,t:87	g:91,t:89	g:93,t:87	g:93,t:89	g:93,t:91
(Intercept)	−1.49	−1.51	1.09	−1.23	1.43	−0.06
	(1.53)	(1.60)	(1.28)	(1.59)	(1.37)	(2.33)
High School	0.71	0.52	0.22	0.32	−0.06	0.69
	(0.90)	(0.98)	(1.12)	(1.03)	(1.17)	(1.40)
College	1.27	1.03	1.64	1.10	1.07	1.76
	(1.36)	(1.45)	(1.82)	(1.47)	(1.93)	(2.92)
Black	0.74	0.83	−0.41	0.65	−0.49	−1.32
	(0.72)	(0.76)	(0.79)	(0.78)	(0.84)	(1.02)
White	0.44	0.39	0.12	0.28	−0.03	−1.08
	(0.62)	(0.65)	(0.69)	(0.68)	(0.74)	(0.89)
Female	0.07	0.20	−1.17	0.20	−1.18	0.22
	(0.89)	(0.96)	(0.94)	(0.98)	(0.99)	(1.88)
IFE	1.98*	2.02*	1.70*	1.99*	1.75*	1.76*
	(0.34)	(0.35)	(0.37)	(0.36)	(0.39)	(0.63)
N	2721	2567	2567	2434	2434	2434
Weak IV F-stat	24.33	23.16	16.64	22.55	15.39	6.11

(b) Post-treatment periods

	g:89,t:89	g:89,t:91	g:89,t:93	g:91,t:91	g:91,t:93	g:93,t:93
(Intercept)	−1.48	−0.37	−0.68	2.47	2.90	−0.79
	(2.39)	(3.04)	(3.60)	(2.06)	(2.52)	(3.46)
High School	1.10	1.33	2.56	0.58	1.62	1.77
	(1.46)	(1.98)	(2.34)	(1.75)	(2.15)	(2.07)
College	3.40	6.20*	7.24*	3.64	4.02	1.27
	(2.16)	(2.83)	(3.35)	(2.89)	(3.54)	(4.33)
Black	1.00	−0.69	−0.92	−2.18	−2.81	−1.55
	(1.13)	(1.49)	(1.77)	(1.27)	(1.55)	(1.51)
White	0.78	−0.47	−0.83	−1.13	−1.66	−1.58
	(0.97)	(1.30)	(1.54)	(1.11)	(1.36)	(1.32)
Female	−0.83	−1.39	−2.31	−1.86	−2.90	0.14
	(1.42)	(1.89)	(2.24)	(1.49)	(1.83)	(2.78)
IFE	2.73*	3.30*	3.91*	2.32*	2.92*	2.58*
	(0.53)	(0.68)	(0.81)	(0.58)	(0.71)	(0.93)
N	2567	2434	2434	2434	2434	2434
Weak IV F-stat	23.16	22.55	22.55	15.39	15.39	6.11

* $p < 0.05$

Notes: The table contains estimates of the interactive fixed effects model for untreated potential outcomes using AFQT as the covariate whose effect on untreated potential outcomes does not change over time and includes other covariates in the model (i.e., allowing other covariates to have time varying effects). Columns correspond to estimates for particular groups and particular time periods. For example, the first column in panel (a), which is labeled "g:89,t:87" is for the group of workers who were displaced in 1989 in the year 1987. The rows labeled "IFE" report the estimated value of $F^*_{g,t}$. The row labeled "Weak IV F-stat" report the F-statistic from the first stage regression of the endogenous regressors on AFQT. Some F-statistics are the same because the first stage is the same across some groups/time periods.

Table SA-7: IFE Model Estimates using Education as Covariate with Time Invariant Effect and Additional Covariates

(a) Pre-treatment periods

	g:89,t:87	g:91,t:87	g:91,t:89	g:93,t:87	g:93,t:89	g:93,t:91
(Intercept)	−2.39	−2.38	−0.42	−2.72	0.13	−0.97
	(1.38)	(1.42)	(1.20)	(1.58)	(1.21)	(1.47)
Black	0.91	0.96	−0.63	0.75	−0.65	−1.23
	(0.81)	(0.83)	(0.97)	(0.91)	(0.98)	(1.17)
White	0.49	0.42	−0.06	0.26	−0.18	−1.13
	(0.71)	(0.73)	(0.86)	(0.80)	(0.87)	(1.04)
Female	0.82	0.88	−0.12	1.14	−0.40	1.27
	(0.74)	(0.77)	(0.75)	(0.87)	(0.75)	(0.97)
IFE	2.31*	2.30*	2.19*	2.39*	2.11*	2.13*
	(0.25)	(0.25)	(0.22)	(0.28)	(0.22)	(0.24)
N	2721	2567	2567	2434	2434	2434
Sargan p-value	0.676	0.832	0.600	0.872	0.572	0.837
Weak IV F-stat	30.37	28.99	34.24	24.62	32.87	29.49

(b) Post-treatment periods

	g:89,t:89	g:89,t:91	g:89,t:93	g:91,t:91	g:91,t:93	g:93,t:93
(Intercept)	−5.67*	−9.83*	−9.55*	−0.70	0.94	0.76
	(2.57)	(4.17)	(4.74)	(2.11)	(2.42)	(1.84)
Black	1.48	−0.10	−0.26	−2.62	−3.15	−1.46
	(1.51)	(2.39)	(2.72)	(1.72)	(1.96)	(1.46)
White	0.85	−0.62	−0.92	−1.52	−1.94	−1.45
	(1.32)	(2.11)	(2.39)	(1.51)	(1.73)	(1.29)
Female	1.82	4.27	3.58	0.43	−0.83	0.11
	(1.40)	(2.29)	(2.60)	(1.32)	(1.51)	(1.21)
IFE	3.86*	5.67*	6.35*	3.37*	3.86*	2.53*
	(0.45)	(0.75)	(0.85)	(0.38)	(0.44)	(0.29)
N	2567	2434	2434	2434	2434	2434
Sargan p-value	0.940	0.553	0.875	0.585	0.944	0.295
Weak IV F-stat	28.99	24.62	24.62	32.87	32.87	29.49

* $p < 0.05$

Notes: The table contains estimates of the interactive fixed effects model for untreated potential outcomes using educational attainment (this amounts to dummy variables indicating being a college graduate or being a high school graduate) as the covariates whose effects on untreated potential outcomes do not change over time and includes other covariates in the model (i.e., allowing other covariates to have time varying effects). Columns correspond to estimates for particular groups and particular time periods. For example, the first column in panel (a), which is labeled "g:89,t:87" is for the group of workers who were displaced in 1989 in the year 1987. The rows labeled "IFE" report the estimated value of $F^*_{g,t}$. The row labeled "Sargan p-value" provides the p-value from an over-identification test. The row labeled "Weak IV F-stat" report the F-statistic from the first stage regression of the endogenous regressors on educational attainment. Some F-statistics are the same because the first stage is the same across some groups/time periods.

Table SA-8: First Stage Regression Results by Group and Time Period

(a) Pre-Treatment Periods

	g:89,t:87	g:91,t:87	g:91,t:89	g:93,t:87	g:93,t:89	g:93,t:91
(Intercept)	1.904*	1.875*	1.376*	1.859*	1.313*	1.345*
	(0.268)	(0.280)	(0.334)	(0.292)	(0.350)	(0.415)
AFQT	0.039*	0.039*	0.044*	0.039*	0.045*	0.045*
	(0.005)	(0.005)	(0.006)	(0.005)	(0.006)	(0.007)
N	2721	2567	2567	2434	2434	2434
R^2	0.025	0.026	0.023	0.025	0.023	0.017

(b) Post-Treatment Periods

	g:89,t:89	g:89,t:91	g:89,t:93	g:91,t:91	g:91,t:93	g:93,t:93
(Intercept)	1.875*	1.859*	1.859*	1.313*	1.313*	1.345*
	(0.280)	(0.292)	(0.292)	(0.350)	(0.350)	(0.415)
AFQT	0.039*	0.039*	0.039*	0.045*	0.045*	0.045*
	(0.005)	(0.005)	(0.005)	(0.006)	(0.006)	(0.007)
N	2567	2434	2434	2434	2434	2434
R^2	0.026	0.025	0.025	0.023	0.023	0.017

Notes: This table contains results from the first stage regression of the change in earnings over time (in post treatment time periods for group g, because we allow for two years of anticipation effects, this is the change in earnings from period $g-6$ to $g-4$; in pre-treatment periods, it is the change in earnings from $t-4$ to $t-2$) among non-displaced and not-yet-displaced workers on AFQT scores. Some columns are identical because the first stage is the same across some groups/time periods. Standard errors are reported in parentheses, and a $*$ indicates a p-value less than 0.05.

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