Supplementary Appendix for "Local Intergenerational Elasticities"

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April 15, 2019

This supplementary appendix contains additional details about our estimates of the Adjusted Local Intergenerational Elasticity. In the main text, we showed that

$$\widehat{ALIGE}(t) = \left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)' \widehat{\frac{\partial\beta(t)}{\partial t}}$$

We proposed estimating $\partial\beta(t)/\partial t$ (as well as $\beta(t)$) by regressing Y on X and the interaction of $(T-t)X$ using a local regression that is weighted by how "close" individual values of T are to t. This is a local linear smooth coefficients model, and in the paper we note that our estimates can be written as

$$\begin{pmatrix} \hat{\beta}(t) \\ \widehat{\frac{\partial\beta(t)}{\partial t}} \end{pmatrix} = (\mathbf{X}'\mathbf{K}(t)\mathbf{X})^{-1}\mathbf{X}'\mathbf{K}(t)\mathbf{y}$$

with explicit expressions for each term given in the main text. These estimates are key building blocks for our estimates of the ALIGE. However, due to space limitations, we only reported estimates of the ALIGE itself (as a function of t) in the main text. Here, we report our estimates for $\beta(t)$ and $\partial\beta(t)/\partial t$. Table 1 and Table 2 contain our estimates. We report estimates only for $t \approx 9.9$, $t \approx 10.8$, and $t \approx 11.8$ which are the smallest, middle, and largest values of parents' income for which we estimated ALIGE in the paper (which correspond to parents' income of \$20,000, \$50,273, and \$140,000.[1]

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[1]In the paper, we estimated the ALIGE at 20 different values of t and reported the results as a figure; showing the

Table 1: Coefficients for Covariates (i.e. $\beta(t)$)

Log Parent's Income:	**9.9**	**10.8**	**11.8**
Constant	10.206	10.322	10.630
	(0.062)	(0.059)	(0.107)
Head Male	-0.063	0.100	0.349
	(0.084)	(0.046)	(0.181)
Head White	0.165	0.269	0.308
	(0.071)	(0.040)	(0.159)
Head Veteran	-0.091	-0.007	0.019
	(0.105)	(0.023)	(0.055)
Male	-0.073	0.033	-0.019
	(0.078)	(0.020)	(0.052)
Year Born	-0.012	-0.012	-0.010
	(0.004)	(0.001)	(0.002)
HS Educ.	0.103	0.223	0.046
	(0.084)	(0.030)	(0.112)
Head College Educ.	0.721	0.357	0.183
	(0.131)	(0.041)	(0.107)

Notes: The table shows the estimated coefficients using the local linear smooth coefficients model discussed in the paper. The standard errors are computed using the wild bootstrap with 500 iterations.

The results in Table 1 are easy to interpret. For simplicity, here we discuss the effects of parents' race and parents' having a college education (overall, in our application, these are the two most important observed characteristics). Across all values of parents' income, the "head" of the family being white increases child's income. These effects are largest at the highest levels of parents' income. Similarly, across all values of parents' income, the head of the family having a college education increases child's income. These effects tend to be somewhat larger than the effects of race on child's income. Interestingly, however, the pattern is somewhat different. The effects of head attending college are largest for parents with the lowest income and smallest for parents' with the highest income.

The estimates of the derivatives of our parameters are provided in Table 2. These are the parameters that show up in our estimates of the ALIGE; however, their interpretation is more subtle

17 additional columns only makes the results in the tables below more difficult to understand.

Table 2: Derivatives of Coefficients for Covariates (i.e. $\partial\beta(t)/\partial t$)

Log Parent's Income:	**9.9**	**10.8**	**11.8**
Head Male	0.130	0.488	0.366
	(0.102)	(0.089)	(0.195)
Head White	0.238	0.000	0.211
	(0.097)	(0.091)	(0.183)
Head Veteran	0.111	-0.016	0.038
	(0.125)	(0.052)	(0.074)
Male	0.146	0.018	-0.062
	(0.096)	(0.049)	(0.069)
Year Born	0.000	-0.002	0.004
	(0.005)	(0.002)	(0.004)
HS Educ.	0.203	0.040	-0.206
	(0.113)	(0.069)	(0.134)
Head College Educ.	-0.340	-0.105	-0.139
	(0.149)	(0.084)	(0.133)

Notes: The table shows the derivatives of estimated coefficients using the local linear smooth coefficients model in the paper. These parameters are used to compute the ALIGE that is the main object of interest in the paper. The standard errors are computed using the wild bootstrap with 500 iterations.

than the interpretation of the parameters in Table 1. These parameters should be interpreted as how the effect of one particular covariate changes with small changes in parents' income. For example, although the effect of the family head having a college education is positive and large at low values of parents' income (as reported in Table 1), the reported parameter in Table 2 is negative and large. This is because the effect of the family head attending college is estimated to be decreasing in parents' income.