Homework 2 Solutions

Extra Question 1

```
# load data
data(Star, package="Ecdat")

# limit data to boys in small or regular class
data <- subset(Star,
               classk %in% c("small.class", "regular") & sex=="boy")

# part (a)
att_a <- mean(subset(data, classk == "small.class")$tmathssk) -
  mean(subset(data, classk=="regular")$tmathssk)
att_a
```

```
[1] 13.67522
```

```
# part (b)
data <- droplevels(data)                    # drop extra factors
X <- model.matrix(~classk, data=data) # get data matrix
Y <- as.matrix(data$tmathssk)          # get outcome
bet <- solve(t(X)%*%X)%*%t(X)%*%Y     # estimate beta
att_b <- bet[2]                             # report coefficient on small class
att_b
```

```
[1] 13.67522
```

```
# part (c)
X <- model.matrix(~classk + totexpk + freelunk, data=data) # X w/ extra vars
bet <- solve(t(X)%*%X)%*%t(X)%*%Y                          # estimate beta
att_c <- bet[2]                                              # report coef. on small
att_c
```

```
[1] 13.42333
```

The results from parts (a) and (b) are exactly identical. The result from part (c) is similar, but not exactly the same — this is exactly what we would expect.

Extra Question 2

a)

$$
\begin{aligned}
ATE &= \mathbb{E}[Y(1) - Y(0)] \\
&= \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] \\
&= \mathbb{E}\Big[\mathbb{E}[Y(1)|X]\Big] - \mathbb{E}\Big[\mathbb{E}[Y(0)|X]\Big] \\
&= \mathbb{E}\Big[\mathbb{E}[Y(1)|X, D=1]\Big] - \mathbb{E}\Big[\mathbb{E}[Y(0)|X, D=0]\Big] \\
&= \mathbb{E}\Big[\mathbb{E}[Y|X, D=1]\Big] - \mathbb{E}\Big[\mathbb{E}[Y|X, D=0]\Big]
\end{aligned}
$$

where the first equality is the definition of ATE, the second equality pushes the expectation through the difference, the third equality holds by the law iterated expectations, the fourth equality holds by unconfoundedness, and the last equality holds because $Y = Y(1)$ among the treated group and $Y = Y(0)$ among the untreated group. This shows that ATE is identified.

b) In class, we showed that $ATT = \mathbb{E}[Y|D=1] - \mathbb{E}[\mathbb{E}[Y|X, D=0]|D=1]$. These are notably different. The expression for ATE takes the $\mathbb{E}[Y|X, D=1]$ (the mean of Y conditional on X among the treated group) and averages it over the distribution of X for the entire population and then subtracts $\mathbb{E}[Y|X, D=0]$ (the mean of Y conditional on X among the untreated group) averaged over the population distribution of X.

In contrast, ATT compares the mean of Y among the treated group to $\mathbb{E}[Y|X, D=0]$ where this is averaged over the distribution of X *among the treated group*.

An intuition for why ATE involves averaging over the distribution of X for the entire population is that ATE is the average treatment effect for the entire population.

c) We have that

$$
\begin{aligned}
Y_i &= Y_i(0) + D_i(Y_i(1) - Y_i(0)) \\
&= X_i'\beta + e_i + \alpha D_i \\
&= \alpha D_i + X_i'\beta + e_i
\end{aligned}
$$

where the first equality holds by writing observed outcomes in terms of potential outcomes, the second equality uses the model for untreated potential outcomes and treatment effect homogeneity, and the last equality rearranges terms.

Furthermore, unconfoundedness implies that $\mathbb{E}[e|X, D] = 0$ which implies that α can be estimated from the regression of Y on D and X.

d) This is exactly the same regressions as we talked about in class after we had identified the ATT. This should not be surprising because, if we restrict the effect of participating in the treatment to be the same across all units, then $ATT = ATE = \alpha$.