Extra Practice Questions for Midterm 1, Solutions

1. (a) What would be the result from running the following code?

```
all( c(1,2,3,4,5) > 0)
```

Answer: `TRUE`, this checks if all elements of the vector `1,2,3,4,5` are greater than 0. Since they all are, it returns `TRUE`

(b) Consider the following function

```
a_function <- function(n) {
  out <- 0
  for (i in 1:n) {
    out <- out + i^2
  }
  out
}
```

If you run the following code, what will it output?

```
a_function(5)
```

Answer: It will output `55` which comes from adding up $1 + 4 + 9 + 16 + 25$

2. Suppose there are two random variables X and Y.

a) If you know that X and Y are independent, do you know what their covariance is equal to? Explain. If yes, what is the covariance equal to?

Answer: Yes, their covariance is 0. Independent random variables have 0 covariance.

b) If you know that $\text{cov}(X, Y) = 0$, are X and Y independent? Explain.

Answer: Not necessarily, random variables can have 0 covariance without being fully independent.

c) If you know that $\text{cov}(X, Y) = 1$, are X and Y independent? Explain.

Answer: X and Y are not independent in this case. If they were, their covariance would be equal to 0.

3. Suppose that X_1 and X_2 are two random variables such that $\mathbb{E}[X_1] = 0$, $\mathbb{E}[X_2] = 5$, $\text{var}(X_1) = 1$, $\text{var}(X_2) = 10$ and $\text{cov}(X_1, X_2) = -1$. Suppose that $Y = X_1 + X_2$.

a) What is $\mathbb{E}[Y]$?

Answer:

$$
\begin{aligned}
\mathbb{E}[Y] &= \mathbb{E}[X_1 + X_2] \\
&= \mathbb{E}[X_1] + \mathbb{E}[X_2] \\
&= 0 + 5 = 5
\end{aligned}
$$

where the second equality holds because expectations can pass through sums

b) What is $\text{var}(Y)$?

Answer:

$$
\begin{aligned}
\text{var}(Y) &= \text{var}(X_1 + X_2) \\
&= \text{var}(X_1) + \text{var}(X_2) + 2\text{cov}(X_1, X_2) \\
&= 1 + 10 + 2(-1) \\
&= 9
\end{aligned}
$$

4. Consider a random variable Y that is equal to a firm's profits (in thousands of dollars) and another random variable X that is equal to firm's number of employees. Suppose you know that

$$\mathbb{E}[Y|X = x] = 50 + 10x$$

   a) Explain how to interpret $\mathbb{E}[Y|X = x]$.

   Answer: This is the conditional expectation of Y given X takes the particular value x. In the context of the problem, it is the mean profit's of firms that have x number of employees. The value can change for different numbers of employees.

   b) What is $\mathbb{E}[Y|X = 10]$?

   Answer: $\mathbb{E}[Y|X = 10] = 50 + 10(10) = 150$

   c) Suppose that $\text{var}(Y) = 40$, $\mathbb{E}[X] = 30$, and $\text{var}(Y) = 20$, calculate $\mathbb{E}[Y]$.

   Answer:

$$
\begin{aligned}
\mathbb{E}[Y] &= \mathbb{E}[50 + 10X] \\
&= 50 + 10\mathbb{E}[X] \\
&= 50 + 10(30) \\
&= 350
\end{aligned}
$$

5. Suppose that we have a random sample of n observations of X and Y.

   a) Suppose that you want to estimate the covariance between X and Y using the data that we have. Propose an estimator for the covariance. **Hint:** Try using the analogy principle and the expression $\text{cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$.

   Answer:

$$\widehat{\text{cov}}(X, Y) = \frac{1}{n}\sum_{i=1}^{n} X_i Y_i - \left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)\left(\frac{1}{n}\sum_{i=1}^{n} Y_i\right)$$

   b) Alternatively, the definition of covariance is $\text{cov}(X, Y) = \mathbb{E}\left[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])\right]$. Propose an estimator for the covariance based on this expression. Would you expect this to give you the same estimate of the covariance as in part a?

   Answer: To conserve on notation, let $\bar{X}$ and $\bar{Y}$ denote the sample averages of X and Y, respectively. Then,

$$\widehat{\text{cov}}(X, Y) = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})$$