Extra Practice Questions for Midterm 1

1. (a) What would be the result from running the following code?

```
all( c(1,2,3,4,5) > 0)
```

   (b) Consider the following function

```
a_function <- function(n) {
  out <- 0
  for (i in 1:n) {
    out <- out + i^2
  }
  out
}
```

   If you run the following code, what will it output?

```
a_function(5)
```

2. Suppose there are two random variables X and Y.

   a) If you know that X and Y are independent, do you know what their covariance is equal to? Explain. If yes, what is the covariance equal to?

   b) If you know that $\text{cov}(X, Y) = 0$, are X and Y independent? Explain.

   c) If you know that $\text{cov}(X, Y) = 1$, are X and Y independent? Explain.

3. Suppose that X_1 and X_2 are two random variables such that $\mathbb{E}[X_1] = 0$, $\mathbb{E}[X_2] = 5$, $\text{var}(X_1) = 1$, $\text{var}(X_2) = 10$ and $\text{cov}(X_1, X_2) = -1$. Suppose that $Y = X_1 + X_2$.

   a) What is $\mathbb{E}[Y]$?

   b) What is $\text{var}(Y)$?

4. Consider a random variable Y that is equal to a firm's profits (in thousands of dollars) and another random variable X that is equal to firm's number of employees. Suppose you know that

$$\mathbb{E}[Y|X = x] = 50 + 10x$$

   a) Explain how to interpret $\mathbb{E}[Y|X = x]$.

   b) What is $\mathbb{E}[Y|X = 10]$?

   c) Suppose that $\text{var}(Y) = 40$, $\mathbb{E}[X] = 30$, and $\text{var}(Y) = 20$, calculate $\mathbb{E}[Y]$.

5. Suppose that we have a random sample of n observations of X and Y.

   a) Suppose that you want to estimate the covariance between X and Y using the data that we have. Propose an estimator for the covariance. **Hint:** Try using the analogy principle and the expression $\text{cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$.

   b) Alternatively, the definition of covariance is $\text{cov}(X, Y) = \mathbb{E}\left[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])\right]$. Propose an estimator for the covariance based on this expression. Would you expect this to give you the same estimate of the covariance as in part a?